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FLUID MECHANICS AND HYDRAULIC MACHINES

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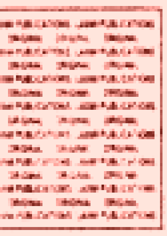

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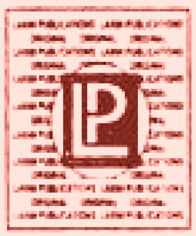

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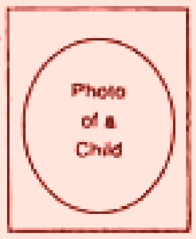

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CHAPTER

Properties of Fluids

▶ 1.1 INTRODUCTION

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

▶ 1.2 PROPERTIES OF FLUIDS

1.2.1 Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

1.2.2 Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically, $$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$\therefore \qquad w = \rho g \qquad \text{...(1.1)}$$

The value of specific weight or weight density (w) for water is 9.81×1000 Newton/m^3 in SI units.

1.2.3 Specific Volume. Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\dfrac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

1.2.4 Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

$$\text{Mathematically,} \quad S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\begin{aligned}\text{Thus weight density of a liquid} &= S \times \text{Weight density of water}\\ &= S \times 1000 \times 9.81 \text{ N/m}^3\\ \text{The density of a liquid} &= S \times \text{Density of water}\\ &= S \times 1000 \text{ kg/m}^3. \qquad \text{...(1.1}A)\end{aligned}$$

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example the specific gravity of mercury is 13.6, hence density of mercury $= 13.6 \times 1000 = 13600$ kg/m^3.

Problem 1.1 *Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.*

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \qquad \left(\because \; 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \text{ N/m}^3. \text{ Ans.}}$$

$$(ii) \text{ Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3. \text{ Ans.}}$$

$$(iii) \text{ Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \qquad \{\because \; \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135. \text{ Ans.}}$$

Problem 1.21 *Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Using equation (1.1.A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(*iii*) *Weight* (W)

We know that specific weight $= \dfrac{\text{Weight}}{\text{Volume}}$

or $w = \dfrac{W}{0.001}$ or $6867 = \dfrac{W}{0.001}$

$\therefore$ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

▶ 1.3 VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and $u + du$ as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ called Tau.

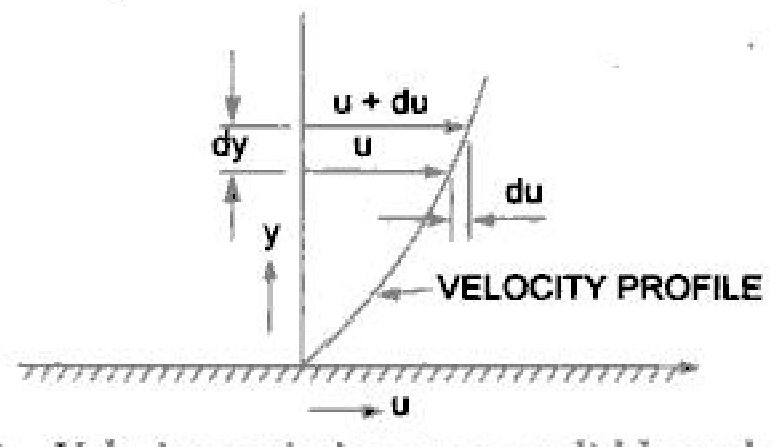

Fig. 1.1 *Velocity variation near a solid boundary.*

Mathematically, $\tau \propto \dfrac{du}{dy}$

or $$\tau = \mu \frac{du}{dy} \quad \text{...(1.2)}$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\dfrac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $$\mu = \frac{\tau}{\left(\dfrac{du}{dy}\right)} \quad \text{...(1.3)}$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

1.3.1 Units of Viscosity. The units of viscosity is obtained by putting the dimensions of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In MKS system, force is represented by kgf and length by metre (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by metre (m).

$\therefore$ MKS unit of viscosity $= \dfrac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \dfrac{\text{dyne-sec}}{\text{cm}^2}$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa = Pascal

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

SI unit of viscosity $= \dfrac{\text{Newton-sec}}{\text{m}^2} = \dfrac{\text{Ns}}{\text{m}^2}$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

The numerical conversion of the unit of viscosity from MKS unit to CGS unit is given below :

$$\frac{\text{one kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ N-sec}}{\text{m}^2} \qquad \{\because \ 1 \text{ kgf} = 9.81 \text{ Newton}\}$$

But one Newton = one kg (mass) $\times$ one $\left(\dfrac{\text{m}}{\text{sec}^2}\right)$ (acceleration)

$$= \frac{(1000 \text{ gm}) \times (100 \text{ cm})}{\text{sec}^2} = 1000 \times 100 \ \frac{\text{gm-cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \qquad \left\{\because \ \text{dyne} = \text{gm} \times \frac{\text{cm}}{\text{sec}^2}\right\}$$

$$\therefore \quad \frac{\text{one kgf-sec}}{\text{m}^2} = 9.81 \times 100000 \ \frac{\text{dyne-sec}}{\text{m}^2} = 9.81 \times 100000 \ \frac{\text{dyne-sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \ \frac{\text{dyne-sec}}{\text{cm}^2} = 98.1 \text{ poise} \qquad \left\{\because \ \frac{\text{dyne-sec}}{\text{cm}^2} = \text{Poise}\right\}$$

Thus for solving numerical problems, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in MKS.

But $\dfrac{\text{one kgf-sec}}{\text{m}^2} = \dfrac{9.81 \text{ Ns}}{\text{m}^2} = 98.1$ poise

$$\therefore \quad \frac{\text{one Ns}}{\text{m}^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise} \quad \text{or} \quad \text{One poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}.$$

Alternate Method. One poise $= \dfrac{\text{dyne} \times \text{s}}{\text{cm}^2} = \left(\dfrac{1 \text{ gm} \times 1 \text{ cm}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{cm}^2}$

But dyne $= 1 \text{ gm} \times \dfrac{1 \text{ cm}}{s^2}$

$\therefore$ One poise $= \dfrac{1 \text{ gm}}{\text{s cm}} = \dfrac{\dfrac{1}{1000} \text{kg}}{\text{s} \dfrac{1}{100} \text{m}}$

$$= \frac{1}{1000} \times 100 \frac{\text{kg}}{\text{sm}} = \frac{1}{10} \frac{\text{kg}}{\text{sm}} \quad \text{or} \quad 1 \frac{\text{kg}}{\text{sm}} = 10 \text{ poise.}$$

Note. (*i*) In SI units second is represented by 's' and not by 'sec'.

(*ii*) If viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units. Sometimes a unit of viscosity as centipoise is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \quad \text{or} \quad 1 \text{ cP} = \frac{1}{100} \text{ P} \qquad [\text{cP} = \text{Centipoise, P} = \text{Poise}]$$

The viscosity of water at 20°C is 0.01 poise or 1.0 centipoise.

1.3.2 Kinematic Viscosity.

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \dfrac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\dfrac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \dfrac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\dfrac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\begin{aligned} \because \text{ Force} &= \text{Mass} \times \text{Acc.} \\ &= \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \end{aligned}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\dfrac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$

Centistoke means $= \dfrac{1}{100}$ stoke.

1.3.3 Newton's Law of Viscosity.

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation (1.2) or as

$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as **Newtonian** fluids and the fluids which do not obey the above relation are called **Non-newtonian** fluids.

1.3.4 Variation of Viscosity with Temperature.

Temperatur affects the viscosity. The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with the increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

(*i*) **For liquids,** $\mu = \mu_0 \left(\dfrac{1}{1 + \alpha t + \beta t^2} \right)$ ...(1.4*A*)

where μ = Viscosity of liquid at t°C, in poise

μ_0 = Viscosity of liquid at 0°C, in poise

α, β = are constants for the liquid

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$.

The equation (1.4 *A*) shows that with the increase of temperature, the viscosity decreases .

(*ii*) **For a gas,** $\mu = \mu_0 + \alpha t - \beta t^2$ ...(1.4*B*)

where for air $\mu_0 = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$.

The equation (1.4 *B*) shows that with the increase of temperature, the viscosity increases.

1.3.5 Types of Fluids.

The fluids may be classified into the following five types :

1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

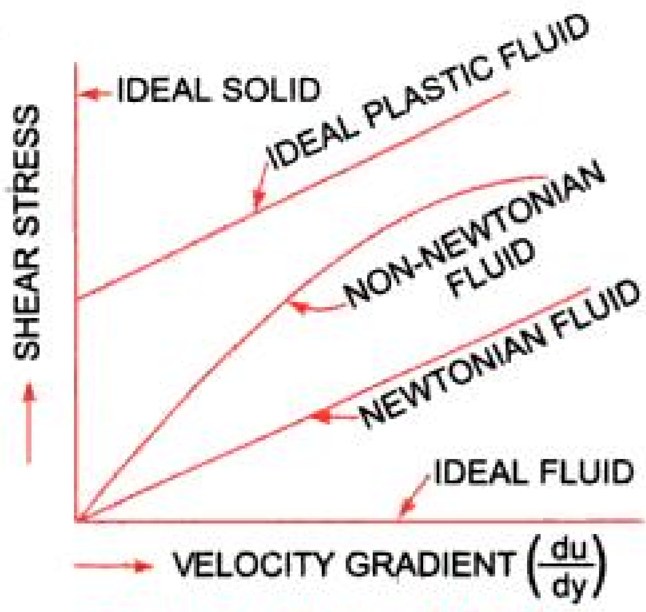

Fig. 1.2 *Types of fluids.*

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

Problem 1.3 *If the velocity distribution over a plate is given by $u = \frac{2}{3} y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.*

Solution. Given : $u = \frac{2}{3} y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2}.\ \textbf{Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2}.\ \textbf{Ans.}$$

Problem 1.4 *A plate, 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.*

Solution. Given :

Distance between plates, $dy = .025$ mm $= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \frac{\text{N}}{\text{m}^2}$.

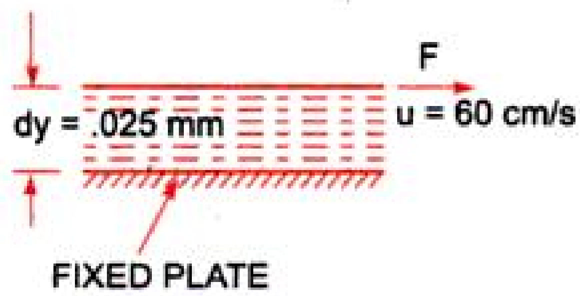

Fig. 1.3

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \frac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \frac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = \mathbf{8.33 \times 10^{-4}\ poise.\ Ans.}$$

Problem 1.5 *A flat plate of area 1.5×10^6 mm^2 is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.*

Solution. Given :

Area of the plate, $A = 1.5 \times 10^6 \text{ mm}^2 = 1.5 \text{ m}^2$

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance between the plates, $dy = 0.15 \text{ mm} = 0.15 \times 10^{-3}$ m

Viscosity $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$.

Using equation (1.2) we have $\tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{.15 \times 10^{-3}} = 266.66 \frac{\text{N}}{\text{m}^2}$

(*i*) ∴ Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 =$ **400 N. Ans.**

(*ii*) Power* required to move the plate at the speed 0.4 m/sec

$= F \times u = 400 \times 0.4 =$ **160 W. Ans.**

Problem 1.6 *Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.*

Solution. Given : $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$

Dia. of shaft, $D = 10 \text{ cm} = 0.1$ m

Distance between shaft and journal bearing,

$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Speed of shaft, $N = 150$ r.p.m.

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785 \text{ m/s}$$

Using equation (1.2), $\tau = \mu \frac{du}{dy}$,

where du = change of velocity between shaft and bearing $= u - 0 = u$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = \mathbf{52.33\ N/m^2.\ Ans.}$$

Problem 1.7 *Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.*

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

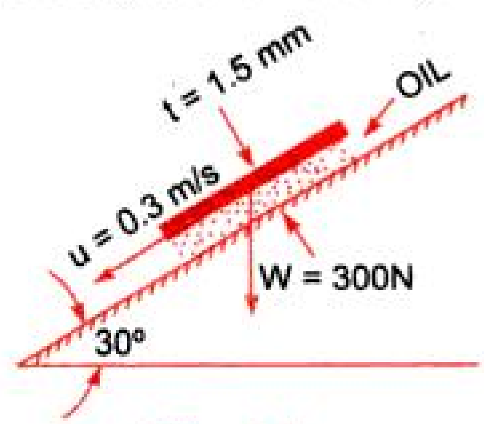

Fig. 1.4

* Power = $F \times u$ N m/s = $F \times u$ W (∵ Nm/s = Watt)

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane = $W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $\tau = \dfrac{F}{\text{Area}} = \dfrac{150}{0.64}$ N/m^2

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity = $u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \textbf{11.7 poise. Ans.}$$

Problem 1.8 *Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.* (A.M.I.E., 1972)

Solution. Given :

Distance between plates, $dy = 1.25 \text{ cm} = 0.0125$ m

Viscosity, $\mu = 14 \text{ poise} = \dfrac{14}{10}$ N s/m^2

Velocity of upper plate, $u = 2.5$ m/sec.

Shear stress is given by equation (1.2) as, $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity between plates = $u - 0 = u = 2.5$ m/sec.

$$dy = 0.0125 \text{ m.}$$

$$\therefore \quad \tau = \frac{14}{10} \times \frac{2.5}{.0125} = \textbf{280 N/m}^2\textbf{. Ans.}$$

Problem 1.9 *The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :*

(i) the dynamic viscosity of the oil in poise, and

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

(A.M.I.E., Winter 1977)

Solution. Given :

Each side of a square plate = 60 cm = 0.60 m

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36$ m^2

Thickness of oil film, $dy = 12.5 \text{ mm} = 12.5 \times 10^{-3}$ m

Velocity of upper plate, $u = 2.5$ m/sec

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\text{ N}}{0.36\text{ m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $$\tau = \mu\frac{du}{dy} \text{ or } \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635\ \frac{\text{Ns}}{\text{m}^2} \qquad \left(\because \frac{1\text{ Ns}}{\text{m}^2} = 10\text{ poise}\right)$$

$= 1.3635 \times 10 =$ **13.635 poise. Ans.**

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950$ kg/m^3

Using the relation, $\nu = \dfrac{\mu}{\rho}$, we get $$\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435\text{ m}^2\text{/sec} = .001435 \times 10^4\text{ cm}^2\text{/s}$$

$=$ **14.35 stokes. Ans.** $\qquad (\because$ cm^2/s = stoke)

Problem 1.10 *Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.*

Solution. Given :

Mass density, $\rho = 981$ kg/m^3

Shear stress, $\tau = 0.2452$ N/m^2

Velocity gradient, $\dfrac{du}{dy} = 0.2$ s

Using the equation (1.2), $$\tau = \mu\frac{du}{dy} \text{ or } 0.2452 = \mu \times 0.2$$

$$\therefore \quad \mu = \frac{0.245}{0.200} = 1.226\text{ Ns/m}^2$$

Kinematic viscosity ν is given by

$$\therefore \quad \nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2}\text{ m}^2\text{/sec}$$

$= 0.125 \times 10^{-2} \times 10^4$ cm^2/s $= 0.125 \times 10^2$ cm^2/s

$= 12.5$ cm^2/s $=$ **12.5 stoke. Ans.** $\qquad (\because$ cm^2/s = stoke)

Problem 1.11. *Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.*

Solution. Given :

Viscosity, $\mu = 0.05$ poise $= \dfrac{0.05}{10}$ N s/m^2

Kinematic viscosity, $\nu = 0.035$ stokes

$= 0.035\ \text{cm}^2/\text{s}$ $\{\because \text{Stoke} = \text{cm}^2/\text{s}\}$

$= 0.035 \times 10^{-4}\ \text{m}^2/\text{s}$

Using the relation $\nu = \frac{\mu}{\rho}$, we get $0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{\rho}$

$$\therefore \quad \rho = \frac{0.05}{10} \times \frac{1}{.035 \times 10^{-4}} = 1428.5\ \text{kg/m}^3$$

$$\therefore \quad \text{Sp. gr. of liquid} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000} = 1.4285 \simeq \textbf{1.43. Ans.}$$

Problem 1.12 *Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.*

Solution. Given :

Kinematic viscosity $\nu = 6$ stokes $= 6\ \text{cm}^2/\text{s} = 6 \times 10^{-4}\ \text{m}^2/\text{s}$

Sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

$$\text{Now sp. gr. of a liquid} = \frac{\text{Density of the liquid}}{\text{Density of water}}$$

or

$$1.9 = \frac{\text{Density of liquid}}{1000}$$

$$\therefore \quad \text{Density of liquid} = 1000 \times 1.9 = 1900\ \frac{\text{kg}}{\text{m}^3}$$

$\therefore$ Using the relation $\nu = \frac{\mu}{\rho}$, we get

$$6 \times 10^{-4} = \frac{\mu}{1900}$$

$$\because \quad \mu = 6 \times 10^{-4} \times 1900 = 1.14\ \text{Ns/m}^2$$

$$= 1.14 \times 10 = \textbf{11.40 poise. Ans.}$$

Problem 1.13 *The velocity distribution for flow over a flat plate is given by $u = \frac{3}{4}y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at y = 0.15 m. Take dynamic viscosity of fluid as 8.6 poise.*

Solution. Given : $u = \frac{3}{4}y - y^2$

$$\therefore \quad \frac{du}{dy} = \frac{3}{4} - 2y$$

At $y = 0.15$, $\frac{du}{dy} = \frac{3}{4} - 2 \times 0.15 = 0.75 - 0.30 = 0.45$

Viscosity, $\mu = 8.5 \text{ poise} = \frac{8.5}{10}\frac{\text{N s}}{\text{m}^2}$ $\left(\because 10 \text{ poise} = 1\frac{\text{Ns}}{\text{m}^2}\right)$

Using equation (1.2), $\tau = \mu \dfrac{du}{dy} = \dfrac{8.5}{10} \times 0.45 \dfrac{\text{N}}{\text{m}^2} = \mathbf{0.3825 \dfrac{N}{m^2}}$. **Ans.**

Problem 1.14 *The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.*

Solution. Given :

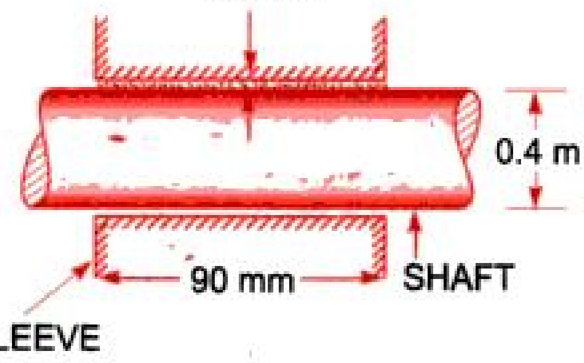

Fig. 1.5

Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{Ns}}{\text{m}^2} = 0.6 \frac{\text{Ns}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

$\therefore$ Shear force on the shaft, F = Shear stress $\times$ Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

$\therefore$ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} = \mathbf{716.48\ W.}$ **Ans.**

Problem 1.15 *If the velocity profile of a fluid over a plate is a parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.*

Solution. Given :

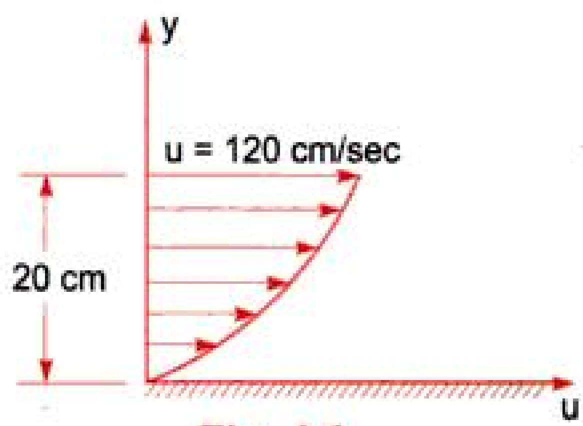

Fig. 1.6

Distance of vertex from plate = 20 cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10}\dfrac{\text{Ns}}{\text{m}^2} = 0.85.$

* Power in S.I. unit $= T * \omega = T \times \dfrac{2\pi N}{60}$ Watt $= \dfrac{2\pi NT}{60}$ Watt

The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \qquad ...(i)$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/sec

(c) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \qquad ...(ii)$$

Boundary condition (c) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \qquad ...(iii)$$

or
$$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (ii) and (iii) for a and b

From equation (iii), $\quad b = -40a$

Substituting this value in equation (ii), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (i),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 =$ **12/s. Ans.**

at $y = 10$ cm, $\quad \left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 =$ **6/s. Ans.**

at $y = 20$ cm, $\quad \left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 =$ **0. Ans.**

Shear Stresses

Shear stress is given by, $\quad \tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2.$

(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2.$

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = \mathbf{0}.$ **Ans.**

Problem 1.16 *A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200 N is applied.* (A.M.I.E., Summer 1980)

Solution. Given : Speed of sleeve, $u_1 = 50$ cm/s

when force, $F_1 = 40$ N.

Let speed of sleeve is u_2 when force, $F_2 = 200$ N.

Using relation $\tau = \mu \frac{du}{dy}$

where τ = Shear stress = $\frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

du = Change of velocity = $u - 0 = u$

dy = Clearance = y

$$\therefore \quad \frac{F}{A} = \mu \frac{u}{y}$$

$$\therefore \quad F = \frac{A\mu u}{y} \alpha u \qquad \{\because \text{ A, } \mu \text{ and } y \text{ are constant}\}$$

$$\therefore \quad \frac{F_1}{u_1} = \frac{F_2}{u_2}$$

Substituting values, we get $\frac{40}{50} = \frac{200}{u_2}$

$$\therefore \quad u_2 = \frac{50 \times 200}{40} = 50 \times 5 = \mathbf{250 \text{ cm/s.}} \textbf{ Ans.}$$

Problem 1.17 *A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m., determine the viscosity of the fluid.* (A.M.I.E., Winter 1979)

Solution. Given :

Diameter of cylinder = 15 cm = 0.15 m

Dia. of outer cylinder = 15.10 cm = 0.151 m

Length of cylinders, L = 25 cm = 0.25 m

Torque, T = 12.0 Nm

Speed, $N = 100$ r.p.m.

Let the viscosity $= \mu$

Tangential velocity of cylinder, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.15 \times 100}{60} = 0.7854$ m/s

Surface area of cylinder, $A = \pi D \times L = \pi \times 0.15 \times 0.25 = .1178 \text{ m}^2$

Now using relation $\tau = \mu \frac{du}{dy}$

where $du = u - 0 = u = .7854$ m/s

$$dy = \frac{0.151 - 0.150}{2} \text{ m} = .0005 \text{ m}$$

$$\tau = \frac{\mu \times .7854}{.0005}$$

$\therefore$ Shear force, $F = \text{Shear stress} \times \text{Area} = \frac{\mu \times .7854}{.0005} \times .1178$

$\therefore$ Torque, $T = F \times \frac{D}{2}$

$$12.0 = \frac{\mu \times .7854}{.0005} \times .1178 \times \frac{.15}{2}$$

$$\therefore \quad \mu = \frac{12.0 \times .0005 \times 2}{.7854 \times .1178 \times .15} = 0.864 \text{ N s/m}^2$$

$= 0.864 \times 10 =$ **8.64 poise. Ans.**

Problem 1.18 *Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if :*

(i) the thin plate is in the middle of the two plane surfaces, and

(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ? Take the dynamic viscosity of glycerine = 8.10×10^{-1} N s/m^2.

Solution. Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1} \text{ N s/m}^2$

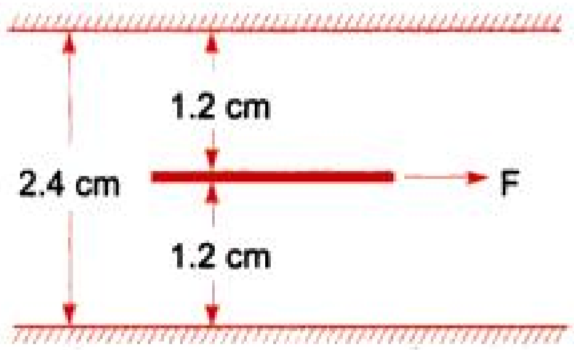

Fig. 1.7 (*a*)

Case I. When the thin plate is in the middle of the two plane surfaces [Refer to Fig. 1.7 (*a*)]

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then $F = F_1 + F_2$

The shear stress (τ_2) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu\left(\frac{du}{dv}\right)_1$$

where du = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\therefore \qquad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area
$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

∴ Shear force, $F_2 = \tau_2 \times \text{A} = 40.5 \times 0.5 = 20.25$ N

∴ Total force, $F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

Case II. When the thin plate is at a distance of 0.8 cm from one of the plane surfaces [Refer to Fig. 1.7 (*b*)].

Let the thin plate is a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface
= 2.4 – 0.8 = 1.6 cm = .016 m
(Neglecting thickness of the plate)

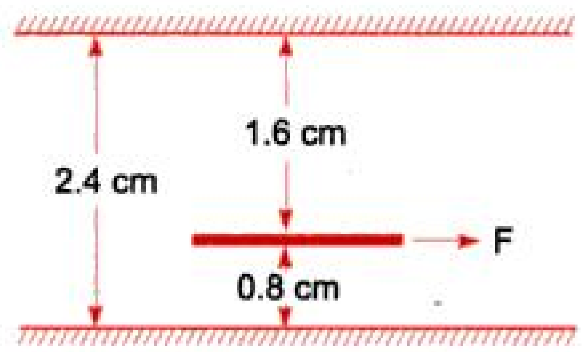

Fig. 1.7 (*b*)

The shear force on the upper side of the thin plate,

F_1 = Shear stress × Area = $\tau_1 \times A$

$$= \mu\left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu\left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

∴ Total force required = $F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

Problem 1.19 *A vertical gap 2.2 cm wide of infinite extent contains a fluid of viscosity 2.0 N s/m² and specific gravity 0.9. A metallic plate 1.2 m × 1.2 m × 0.2 cm is to be lifted up with a constant velocity of 0.15 m/sec, through the gap. If the plate is in the middle of the gap, find the force required. The weight of the plate is 40 N.*

Solution. Given :

Width of gap = 2.2 cm, viscosity, $\mu = 2.0$ N s/m²

Sq. gr. of fluid = 0.9

$\therefore$ Weight density of fluid

$$= 0.9 \times 1000 = 900 \text{ kgf/m}^3 = 900 \times 9.81 \text{ N/m}^3$$

$(\because 1 \text{ kgf} = 9.81 \text{ N})$

Volume of plate $= 1.2 \text{ m} \times 1.2 \text{ m} \times 0.2 \text{ cm}$

$= 1.2 \times 1.2 \times .002 \text{ m}^3 = .00288 \text{ m}^3$

Thickness of plate $= 0.2$ cm

Velocity of plate $= 0.15$ m/sec

Weight of plate $= 40$ N.

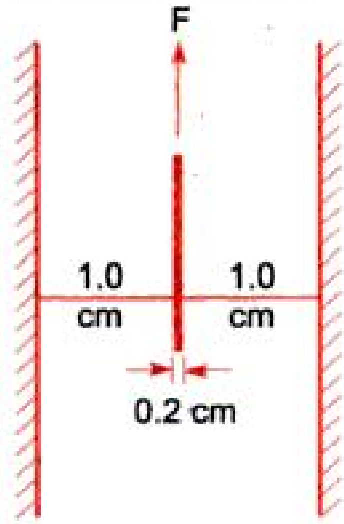

Fig. 1.8

When plate is in the middle of the gap, the distance of the plate of plate from vertical surface, of the gap

$$= \left(\frac{\text{Width of gap} - \text{Thickness of plate}}{2}\right)$$

$$= \frac{(2.2 - 0.2)}{2} = 1 \text{ cm} = .01 \text{ m}.$$

Now the shear force on the left side of the metallic plate,

$$F_1 = \text{Shear stress} \times \text{Area}$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times \text{Area} = 2.0 \times \left(\frac{0.15}{.01}\right) \times 1.2 \times 1.2 \text{ N}$$

$(\because \text{Area} = 1.2 \times 1.2 \text{ m}^2)$

$$= 43.2 \text{ N}.$$

Similarly, the shear force on the right side of the metallic plate,

$$F_2 = \text{Shear stress} \times \text{Area} = 2.0 \times \left(\frac{0.15}{.01}\right) \times 1.2 \times 1.2 = 43.2 \text{ N}$$

$\therefore$ Total shear force $= F_1 + F_2 = 43.2 + 43.2 = 86.4$ N.

In this case the weight of plate (which is acting vertically downward) and upward thrust is also to be taken into account.

The upward thrust = Weight of fluid displaced

$= (\text{Weight density of fluid}) \times \text{Volume of fluid displaced}$

$= 9.81 \times 900 \times .00288$ N

$(\because$ Volume of fluid displaced = Volume of plate = .00288)

$= 25.43$ N.

The net force acting in the downward direction due to weight of the plate and upward thrust

$= \text{Weight of plate} - \text{Upward thrust} = 40 - 25.43 = 14.57$ N

$\therefore$ Total force required to lift the plate up

$= \text{Total shear force} + 14.57 = 86.4 + 14.57 =$ **100.97 N. Ans.**

▶ 1.4 THERMODYNAMIC PROPERTIES

Fluids consist of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role. With the change of pressure and temperature, the gases undergo large variation in density. The relationship between pressure (absolute), specific volume and temperature (absolute) of a gas is given by the equation of state as

$$p\,\forall = RT \text{ or } \frac{p}{\rho} = RT \qquad ...(1.5)$$

where p = Absolute pressure of a gas in N/m^2

$\forall$ = Specific volume = $\frac{1}{\rho}$

R = Gas constant

T = Absolute temperature in °K

ρ = Density of a gas.

1.4.1 Dimension of R. The gas constant, R, depends upon the particular gas. The dimension of R is obtained from equation (1.5) as

$$R = \frac{p}{\rho T}$$

(*i*) In MKS units $$R = \frac{\text{kgf/m}^2}{\left(\frac{\text{kg}}{\text{m}^3}\right)°\text{K}} = \frac{\text{kgf-m}}{\text{kg °K}}$$

(*ii*) In SI units, p is expressed in Newton/m^2 or N/m^2.

$$\therefore \qquad R = \frac{\text{N/m}^2}{\frac{\text{kg}}{\text{m}^3} \times \text{K}} = \frac{\text{N-m}}{\text{kg-K}} = \frac{\text{Joule}}{\text{kg-K}} \qquad [\text{Joule = N-m}]$$

$$= \frac{\text{J}}{\text{kg-K}}$$

For air, $$R \text{ in MKS} = 29.3 \frac{\text{kgf-m}}{\text{kg °K}}$$

$$R \text{ in SI} = 29.3 \times 9.81 \frac{\text{N-m}}{\text{kg°K}} = 287 \frac{\text{J}}{\text{kg-K}}$$

1.4.2 Isothermal Process. If the changes in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (p) and density (ρ) is given by

$$\frac{p}{\rho} = \text{Constant} \qquad ...(1.6)$$

1.4.3 Adiabatic Process. If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^k} = \text{Constant} \qquad ...(1.7)$$

where k = Ratio of specific heat of a gas at constant pressure and constant volume.

= 1.4 for air.

1.4.4 Universal Gas Constant.

Let m = Mass of a gas in kg

$\forall$ = Volume of gas of mass m

p = Absolute pressure

T = Absolute temperature

Then, we have $$p\forall = mRT \quad ...(1.8)$$

where R = Gas constant.

Equation (1.8) can be made universal, *i.e.*, applicable to all gases if it is expressed in **mole-basis**.

Let n = Number of moles in volume of a gas

$\forall$ = Volume of the gas

$$M = \frac{\text{Mass of the gas molecules}}{\text{Mass of a hydrogen atom}}$$

m = Mass of a gas in kg

Then, we have $n \times M = m$.

Substituting the value of m in equation (1.8), we get

$$p\forall = n \times M \times RT \quad ...(1.9)$$

The product $M \times R$ is called universal gas constant and is equal to 848 $\frac{\text{kgf-m}}{\text{kg-mole °K}}$ in MKS units and 8314 J/kg-mole K in SI units.

One kilogram mole is defined as the product of one kilogram mass of the gas and its molecular weight.

Problem 1.20 *A gas weighs 16 N/m³ at 25°C and at an absolute pressure of 0.25 N/mm². Determine the gas constant and density of the gas.*

Solution. Given :

Weight density, $w = 16\ \text{N/m}^2$

Temperature, $t = 25°\text{C}$

$\therefore$ $T = 273 + t = 273 + 25 = 288°\text{K}$

$p = 0.25\ \text{N/mm}^2\ \text{(abs.)} = 0.25 \times 10^6\ \text{N/m}^2 = 25 \times 10^4\ \text{N/m}^2$

(*i*) Using relation $w = \rho g$, density is obtained as

$$\rho = \frac{w}{g} = \frac{16}{9.81} = \mathbf{1.63\ kg/m^3.\ Ans.}$$

(*ii*) Using equation (1.5), $\frac{p}{\rho} = RT$

$$\therefore \quad R = \frac{p}{\rho T} = \frac{25 \times 10^4}{1.63 \times 288} = \mathbf{532.55\ \frac{Nm}{kg\ K}}.\ \mathbf{Ans.}$$

Problem 1.21 *A cylinder of 0.6 m³ in volume contains air at 50°C and 0.3 N/mm² absolute pressure. The air is compressed to 0.3 m³. Find (i) pressure inside the cylinder assuming isothermal process and (ii) pressure and temperature assuming adiabatic process. Take k = 1.4.*

Solution. Given :

Initial volume, $\forall_1 = 0.6\ \text{m}^3$

Temperature $t_1 = 50°C$

$\therefore$ $T_1 = 273 + 50 = 323°K$

Pressure $p_1 = 0.3 \text{ N/mm}^2 = 0.3 \times 10^6 \text{ N/m}^2 = 30 \times 10^4 \text{ N/m}^2$

Final volume $\forall_2 = 0.3 \text{ m}^3$

$k = 1.4$

(*i*) Isothermal process :

Using equation (1.6), $\frac{p}{\rho}$ = Constant or $p\forall$ = Constant.

$$\therefore \quad p_1\forall_1 = p_2\forall_2$$

$$\therefore \quad p_2 = \frac{p_1\forall_1}{\forall_2} = \frac{30 \times 10^4 \times 0.6}{0.3} = 0.6 \times 10^6 \text{ N/m}^2 = \mathbf{0.6 \text{ N/mm}^2. \text{ Ans.}}$$

(*ii*) Adiabatic process :

Using equation (1.7), $\frac{p}{\rho^k}$ = Constant or $p\,\forall^k$ = Constant

$$\therefore \quad p_1\forall_1^k = p_2\forall_2^k.$$

$$\therefore \quad p_2 = p_1\frac{\forall_1^k}{\forall_2^k} = 30 \times 10^4 \times \left(\frac{0.6}{0.3}\right)^{1.4} = 30 \times 10^4 \times 2^{1.4}$$

$$= 0.791 \times 10^6 \text{ N/m}^2 = \mathbf{0.791 \text{ N/mm}^2. \text{ Ans.}}$$

For temperature, using equation (1.5), we get

$$p\forall = RT \text{ and also } p\,\forall^k = \text{Constant}$$

$$\therefore \quad p = \frac{RT}{\forall} \text{ and } \frac{RT}{\forall} \times \forall^k = \text{Constant}$$

or $$RT\forall^{k-1} = \text{Constant}$$

or $$T\,\forall^{k-1} = \text{Constant} \qquad \{\because\ R \text{ is also constant}\}$$

$$\therefore \quad T_1\,\forall_1^{k-1} = T_2\,\forall_2^{k-1}$$

$$\therefore \quad T_2 = T_1\left(\frac{\forall_1}{\forall_2}\right)^{k-1} = 323\left(\frac{0.6}{0.3}\right)^{1.4-1.0} = 323 \times 2^{0.4} = 426.2°K$$

$$\therefore \quad t_2 = 426.2 - 273 = \mathbf{153.2°C. \text{ Ans.}}$$

Problem 1.22 *Calculate the pressure exerted by 5 kg of nitrogen gas at a temperature of 10°C if the volume is 0.4 m³. Molecular weight of nitrogen is 28. Assume, ideal gas laws are applicable.*

Solution. Given :

Mass of nitrogen = 5 kg

Temperature, $t = 10°C$

$\therefore$ $T = 273 + 10 = 283°K$

Volume of nitrogen, $\forall = 0.4 \text{ m}^3$

Molecular weight = 28

Using equation (1.9), we have $p\forall = n \times M \times RT$

where $M \times R$ = Universal gas constant = $8314 \dfrac{\text{N-m}}{\text{kg-mole °K}}$

and one kg-mole = (kg-mass) × Molecular weight = (kg-mass) × 28

$$\therefore \qquad \text{R for nitrogen} = \frac{8314}{28} = 296.9 \ \frac{\text{N-m}}{\text{kg °K}}$$

The gas laws for nitrogen is $p\forall = mRT$, where R = Characteristic gas constant

or $$p \times 0.4 = 5 \times 296.9 \times 283$$

$$\therefore \qquad p = \frac{5 \times 296.9 \times 283}{0.4} = 1050283.7 \text{ N/m}^2 = \mathbf{1.05 \ N/mm^2}. \ \textbf{Ans.}$$

▶ 1.5 COMPRESSIBILITY AND BULK MODULUS

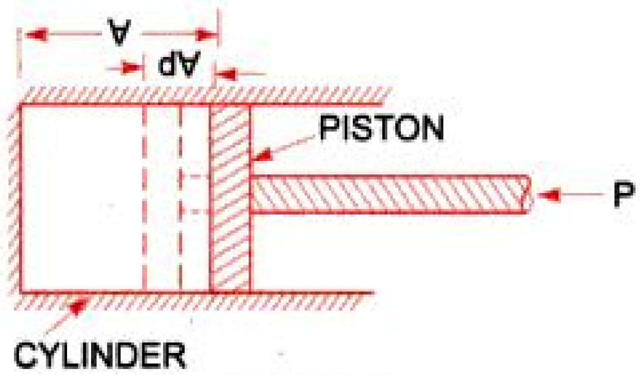

Fig. 1.9

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in Fig. 1.9.

Let $\forall$ = Volume of a gas enclosed in the cylinder

p = Pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

Then increase in pressure $= dp$ kgf/m²

Decrease in volume $= d\forall$

$$\therefore \quad \text{Volumetric strain} = -\frac{d\forall}{\forall}$$

– ve sign means the volume decreases with increase of pressure.

$$\therefore \quad \text{Bulk modulus} \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\dfrac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall \qquad \text{...(1.10)}$$

$$\text{Compressibility is given by} = \frac{1}{K} \qquad \text{...(1.11)}$$

Relationship between Bulk Modulus (K) and Pressure (p) for a Gas

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of compression are as :

(*i*) **For Isothermal Process.** Equation (1.6) gives the relationship between pressure (p) and density (ρ) of a gas as

$$\frac{p}{\rho} = \text{Constant}$$

or $$p\forall = \text{Constant} \qquad \left\{\because\ \forall = \frac{1}{\rho}\right\}$$

Differentiating this equation, we get (p and $\forall$ both are variables)

$$pd\forall + \forall dp = 0 \quad \text{or} \quad pd\forall = -\forall dp \quad \text{or} \quad p = \frac{-\forall dp}{d\forall}$$

Substituting this value in equation (1.10), we get

$$K = p \qquad ...(1.12)$$

(*ii*) **For Adiabatic Process.** Using equation (1.7) for adiabatic process

$$\frac{p}{\rho^k} = \text{Constant or } p\,\forall^k = \text{Constant}$$

Differentiating, we get $pd(\forall^k) + \forall^k(dp) = 0$

or $$p \times k \times \forall^{k-1}\, d\forall + \forall^k\, dp = 0$$

or $$pkd\forall + \forall dp = 0 \qquad \text{[Cancelling } \forall^{k-1} \text{ to both sides]}$$

or $$pkd\forall = -\forall dp \quad \text{or} \quad pk = -\frac{\forall dp}{d\forall}$$

Hence from equation (1.10), we have

$$K = pk \qquad ...(1.13)$$

where K = Bulk modulus and k = Ratio of specific heats.

Problem 1.23 *Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15 per cent.*

Solution. Given :

Initial pressure $= 70\ \text{N/cm}^2$

Final pressure $= 130\ \text{N/cm}^2$

$\therefore$ dp = Increase in pressure $= 130 - 70 = 60\ \text{N/cm}^2$

Decrease in volume $= 0.15\%$

$$\therefore \qquad -\frac{d\forall}{\forall} = +\frac{0.15}{100}$$

Bulk modulus, K is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{60\ \text{N/cm}^2}{\frac{.15}{100}} = \frac{60 \times 100}{.15} = \mathbf{4 \times 10^4\ N/cm^2.\ Ans.}$$

Problem 1.24 *What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m³ at 80 N/cm² pressure to a volume of 0.0124 m³ at 150 N/cm² pressure ?*

Solution. Given :

Initial volume, $\forall = 0.0125\ \text{m}^3$

Final volume $= 0.0124\ \text{m}^3$

$\therefore$ Decrease in volume, $d\forall = .0125 - .0124 = .0001\ \text{m}^3$

$$\therefore \qquad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

Initial pressure $= 80\ \text{N/cm}^2$

Final pressure $= 150\ \text{N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70\ \text{N/cm}^2$

Bulk modulus is given by equation (1.10) as

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} = \frac{70}{\dfrac{.0001}{.0125}} = 70 \times 125\ \text{N/cm}^2$$

$$= \mathbf{8.75 \times 10^3\ N/cm^2.\ Ans.}$$

1.6 SURFACE TENSION AND CAPILLARITY

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrance under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule A is zero. But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule B is acting in the downward direction. The molecule C, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrance under tension.

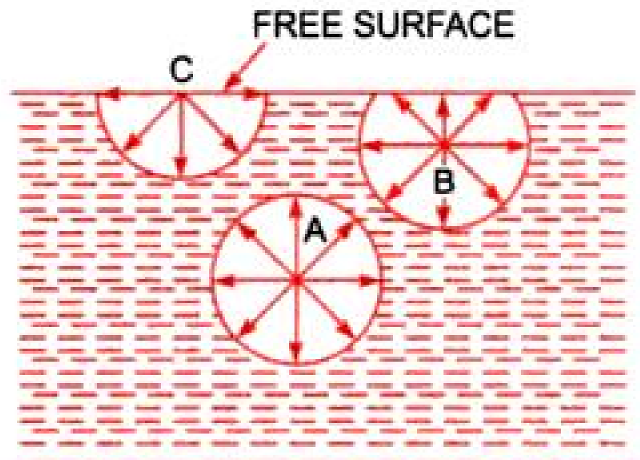

Fig. 1.10 *Surface tension.*

1.6.1 Surface Tension on Liquid Droplet.

Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(*i*) tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 1.11 (*b*) and this is equal to

$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$

(*ii*) pressure force on the area $\frac{\pi}{4} d^2$ and $= p \times \frac{\pi}{4} d^2$ as shown in Fig. 1.11 (*c*). These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

or
$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d} \qquad \text{...(1.14)}$$

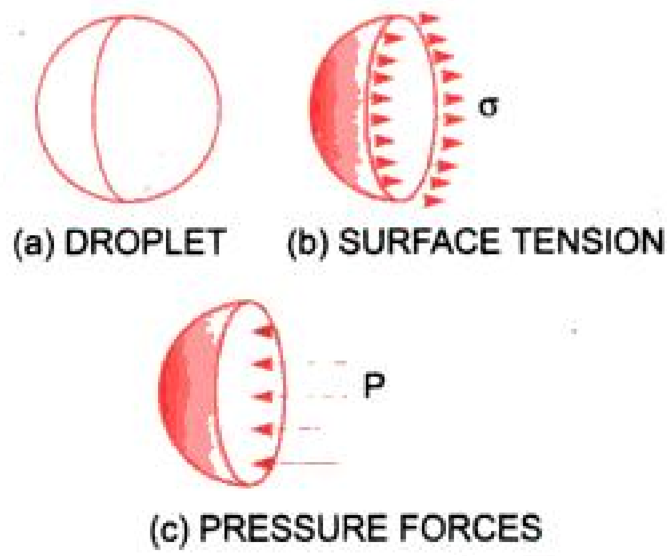

Fig. 1.11 *Forces on droplet.*

Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

1.6.2 Surface Tension on a Hollow Bubble. A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$\therefore$
$$p = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \qquad \text{...(1.15)}$$

1.6.3 Surface Tension on a Liquid Jet. Consider a liquid jet of diameter '*d*' and length 'L' as shown in Fig. 1.12.

Let p = Pressure intensity inside the liquid jet above the outside pressure

σ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure $= p \times$ area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L.$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$\therefore$
$$p = \frac{\sigma \times 2L}{L \times d} \qquad \text{...(1.16)}$$

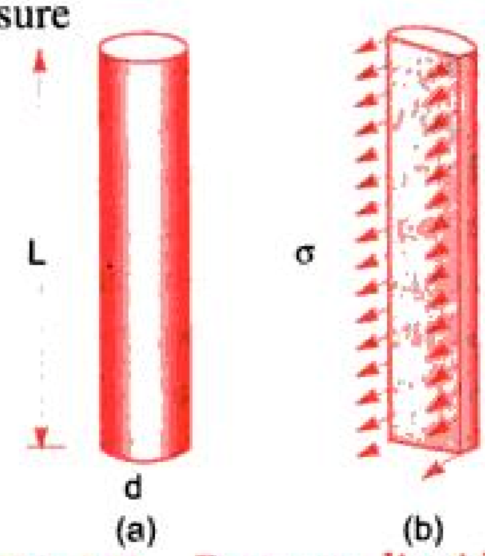

Fig. 1.12 *Forces on liquid jet.*

Problem 1.25 *The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.*

Solution. Given :

Surface tension, $\sigma = 0.0725$ N/m

Pressure intensity, p in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

Let d = dia. of the droplet

Using equation (1.14), we get $p = \dfrac{4\sigma}{d}$ or $0.02 \times 10^4 = \dfrac{4 \times 0.0725}{d}$

$$\therefore \quad d = \frac{4 \times 0.0725}{0.02 \times (10)^4} = .00145 \text{ m} = .00145 \times 1000 = \textbf{1.45 mm. Ans.}$$

Problem 1.26 *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.*

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5$ N/m²

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \textbf{0.0125 N/m. Ans.}$$

Problem 1.27 *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3}$ m

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725$ N/m

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 =$ **11.045 N/cm². Ans.**

1.6.4 Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

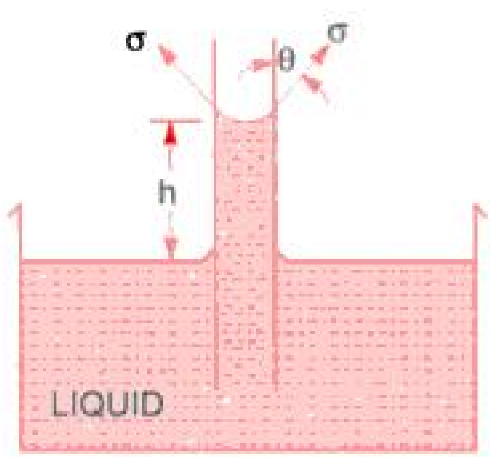

Fig. 1.13 Capillary rise.

Expression for Capillary Rise. Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube $\times h) \times \rho \times g$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad ...(1.17)$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta \quad ...(1.18)$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d} \quad ...(1.19)$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \quad ... (1.20)$$

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \; \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \quad h = \frac{4\sigma\cos\theta}{\rho g d} \quad ...(1.21)$$

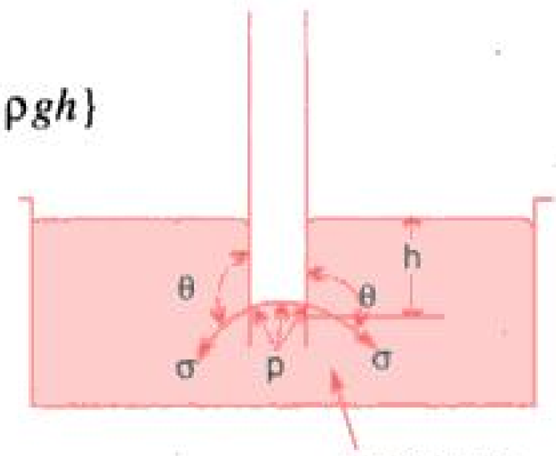

Fig. 1.14

Value of θ for mercury and glass tube is 128°.

Problem 1.28 *Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.*

Solution. Given :

Dia. of tube,	$d = 2.5\text{ mm} = 2.5 \times 10^{-3}$ m
Surface tenstion, σ for water	= 0.0725 N/m
σ for mercury	= 0.52 N/m
Sp. gr. of mercury	= 13.6

$\therefore$ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

(*a*) **Capillary rise for water (θ = 0)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118 \text{ m} = \mathbf{1.18 \text{ cm. Ans.}}$

(*b*) **For mercury**

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004 \text{ m} = -\mathbf{0.4 \text{ cm. Ans.}}$

The negative sign indicates the capillary depression.

Problem 1.29 *Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 1.30°. Take density of water at 20°C as equal to 998 kg/m³.* (U.P.S.C. Engg. Exam., 1974)

Solution. Given :

Dia of tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

The capillary effect (*i.e.*, capillary rise or depression) is given by equation (1.20) as

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tenstion in kgf/m

θ = angle of contact, and ρ = density

(*i*) **Capillary effect for water**

$\sigma = 0.073575 \text{ N/m}, \theta = 0°$

$\rho = 998 \text{ kg/m}^3$ at 20°C

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51 \text{ mm. Ans.}}$$

(*ii*) **Capillary effect for mercury**

$\sigma = 0.51 \text{ N/m}, \theta = 130°$ and

$\rho = \text{sp. gr.} \times 1000 = 13.6 \times 1000 = 13600 \text{ kg/m}^2$

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \mathbf{-2.46 \text{ mm. Ans.}}$$

The negative sign indicates the capillary depression.

Problem 1.30 *The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.*

Solution. Given :

Capillary rise, $h = 0.2 \text{ mm} = 0.2 \times 10^3 \text{ m}$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

Let dia. of tube $= d$

The angle θ for water $= 0$

Density (ρ) for water $= 1000 \text{ kg/m}^3$

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.0725}{1000 \times 9.81 \times .2 \times 10^{-3}} = 0.148 \text{ m} = \mathbf{14.8 \text{ cm. Ans.}}$$

Thus minimum diameter of the tube should be 14.8 cm.

Problem 1.31 *Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.* (Converted to SI Units, A.M.I.E., Summer 1985)

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.073575 \text{ N/m}$

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5 \text{ cm. Ans.}}$$

Thus minimum diameter of the tube should be 1.5 cm.

Problem 1.32 *An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5 m and it rotates at 200 r.p.m. Calculate the power lost in oil for a sleeve length of 100 mm. The thickness of oil film is 1.0 mm.* [Delhi University, December, 1992 (NS)]

Solution. Given :

Viscosity, $\mu = 5 \text{ poise}$

$$= \frac{5}{10} = 0.5 \text{ N s/m}^2$$

Dia. of shaft, $D = 0.5 \text{ m}$

Speed of shaft, $N = 200 \text{ r.p.m.}$

Sleeve length, $L = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1 \text{ m}$

Thickness of oil film, $t = 1.0 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Tangential velocity of shaft, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60} = 5.235 \text{ m/s}$

Using the relation, $\tau = \mu \frac{du}{dy}$

where, du = Change of velocity = $u - 0 = u = 5.235$ m/s

dy = Change of distance = $t = 1 \times 10^{-3}$ m

$$\therefore \qquad \tau = \frac{0.5 \times 5.235}{1 \times 10^{-3}} = 2617.5 \text{ N/m}^2$$

This is the shear stress on the shaft

$\therefore$ Shear force on the shaft, F = Shear stress × Area = $2617.5 \times \pi D \times L$ $\qquad$ ($\because$ Area = $\pi D \times L$)

$$= 2617.5 \times \pi \times 0.5 \times 0.1 = 410.95 \text{ N}$$

Torque on the shaft, $\qquad T = \text{Force} \times \frac{D}{2} = 410.95 \times \frac{0.5}{2} = 102.74$ Nm

$$\therefore \qquad \text{Power* lost} = T \times \omega \text{ Watts} = T \times \frac{2\pi N}{60} \text{ W}$$

$$= 102.74 \times \frac{2\pi \times 200}{60} = 2150 \text{ W} = \mathbf{2.15 \text{ kW. Ans.}}$$

▶1.7 VAPOUR PRESSURE AND CAVITATION

A change from the liquid state to the gaseous state is known as vaporization. The vaporization (which depends upon the prevailing pressure and temperature condition) occurs because of continuous escaping of the molecules through the free liquid surface.

Consider a liquid (say water) which is confined in a closed vessel. Let the temperature of liquid is 20°C and pressure is atmospheric. This liquid will vaporise at 100°C. When vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as **vapour pressure** of the liquid. Or this is the pressure at which the liquid is converted into vapours.

Again consider the same liquid at 20°C at atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means, the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20°C. Thus a liquid may boil even at ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as **cavitation**.

Hence the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which the liquid is flowing, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and hence the name is cavitation.

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* Power in case of S.I. Unit = $T \times \omega$ or $\frac{2\pi NT}{60}$ Watts or $\frac{2\pi NT}{60{,}000}$ kW. The angular velocity $\omega = \frac{2\pi N}{60}$.

HIGHLIGHTS

1. The weight density or specific weight of a fluid is equal to weight per unit volume. It is also equal to,
$$w = \rho \times g.$$
2. Specific volume is the reciprocal of mass density.
3. The shear stress is proportional to the velocity gradient $\frac{du}{dy}$. Mathematically, $\tau = \mu \frac{du}{dy}$.
4. Kinematic viscosity ν is given by $\nu = \frac{\mu}{\rho}$.
5. Poise and stokes are the units of viscosity and kinematic viscosity respectively.
6. To convert the unit of viscosity from poise to MKS units, poise should be divided by 98.1 and to convert poise into SI units, the poise should be divided by 10. SI unit of viscosity is Ns/m^2 or Pa s, where N/m^2 = Pa = Pascal.
7. For a perfect gas, the equation of state is $\frac{p}{\rho} = RT$

   where R = gas constant and for air = $29.3 \frac{\text{kgf-m}}{\text{kg}\,°\text{K}}$ = 287 J/kg °K.
8. For isothermal process, $\frac{p}{\rho}$ = Constant whereas for adiabatic process, $\frac{p}{\rho^k}$ = constant.
9. Bulk modulus of elasticity is given as $K = \frac{-dp}{\left(\frac{d\forall}{\forall}\right)}$.
10. Compressibility is the reciprocal of bulk modulus of elasticity or $= \frac{1}{K}$.
11. Surface tension is expressed in N/m or dyne/cm. The relation between surface tension (σ) and difference of pressure (p) between the inside and outside of a liquid drop is given as $p = \frac{4\sigma}{d}$

    For a soap bubble, $p = \frac{8\sigma}{d}$.

    For a liquid jet, $p = \frac{2\sigma}{d}$.
12. Capillary rise or fall of a liquid is given by $h = \frac{4\sigma \cos\theta}{wd}$.

    The value of θ for water is taken equal to zero and for mercury equal to 128°.

EXERCISE 1

(A) THEORETICAL PROBLEMS

1. Define the following fluid properties :
   Density, weight density, specific volume and specific gravity of a fluid.
2. Differentiate between : (*i*) Liquids and gases, (*ii*) Real fluids and ideal fluids, (*iii*) Specific weight and specific volume of a fluid.
3. What is the difference between dynamic viscosity and kinematic viscosity ? State their units of measurements.

4. Explain the terms : (*i*) Dynamic viscosity, and (*ii*) Kinematic viscosity. Give their dimensions. (*A.M.I.E., Summer 1988*)
5. State the Newton's law of viscosity and give examples of its application. (*Delhi University, June 1996*)
6. Enunciate Newton's law of viscosity. Explain the importance of viscosity in fluid motion. What is the effect of temperature on viscosity of water and that of air? (*A.M.I.E., Winter 1987*)
7. Define Newtonian and Non-Newtonian fluids.
8. What do you understand by terms : (*i*) Isothermal process, (*ii*) Adiabatic process, and (*iii*) Universal-gas constant.
9. Define compressibility. Prove that compressibility for a perfect gas undergoing isothermal compression is $\frac{1}{p}$ while for a perfect gas undergoing isentropic compression is $\frac{1}{wp}$.
10. Define surface tension. Prove that the relationship between surface tension and pressure inside a droplet of liquid in excess of outside pressure is given by $p = \frac{4\sigma}{d}$.
11. Explain the phenomenon of capillarity. Obtain an expression for capillary rise of a liquid.
12. (*a*) Distinguish between ideal fluids and real fluids. Explain the importance of compressibility in fluid flow. (*A.M.I.E., Summer 1988*)
    (*b*) Define the terms : density, specific volume, specific gravity, vacuum pressure, compressible and incompressible fluids. (*R.G.P. Vishwavidyalaya, Bhopal S 2002*)
13. Define and explain Newton's law of viscosity. (*Delhi University, April 1992*)
14. Convert 1 kg/s-m dynamic viscosity in poise. (*A.M.I.E., Winter 1991*)
15. Why does the viscosity of a gas increases with the increase in temperature while that of a liquid decreases with increase in temperature ? (*A.M.I.E., Winter 1990*)
16. (*a*) How does viscosity of a fluid vary with temperature ?
    (*b*) Cite examples where surface tension effects play a prominent role. (*J.N.T.U., Hyderabad S 2002*)
17. (*i*) Develop the expression for the relation between gauge pressure *P* inside a droplet of liquid and the surface tension.
    (*ii*) Explain the following :
    Newtonian and Non-Newtonian fluids, vapour pressure, and compressibility. (*R.G.P.V., Bhopal S 2001*)

(B) NUMERICAL PROBLEMS

1. One litre of crude oil weighs 9.6 N. Calculate its specific weight, density and specific gravity. (*A.M.I.E., Summer 1986*) [**Ans.** 9600 N/m^3, 978.6 kg/m^3, 0.978]
2. The velocity distribution for flow over a flat plate is given by $u = \frac{3}{2}\, y - y^{3/2}$, where *u* is the point velocity in metre per second at a distance *y* metre above the plate. Determine the shear stress at *y* = 9 cm. Assume dynamic viscosity as 8 poise. (*Nagpur University*) [**Ans.** 0.839 N/m^2]
3. A plate, 0.025 mm distant form a fixed plate, moves at 50 cm/s and requires a force of 1.471 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates in the poise. [**Ans.** 7.357×10^{-4}]
4. Determine the intensity of shear of an oil having viscosity = 1.2 poise and is used for lubrication in the clearance between a 10 cm diameter shaft and its journal bearing. The clearance is 1.0 mm and shaft rotates at 200 r.p.m. [**Ans.** 125.56 N/m^2]
5. Two plates are placed at a distance of 0.15 mm apart. The lower plate is fixed while the upper plate having surface area 1.0 m^2 is pulled at 0.3 m/s. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity 1.5 poise. [**Ans.** 300 N, 89.8 W]
6. An oil film of thickness 1.5 mm is used for lubrication between a square plate of size 0.9 m × 0.9 m and an inclined plane having an angle of inclination 20°. The weight of the square is 392.4 N and it slides down the plane with a uniform velocity of 0.2 m/s. Find the dynamic viscosity of the oil. [**Ans.** 12.42 poise]

7. In a stream of glycerine in motion, at a certain point the velocity gradient is 0.25 metre per sec per metre. The mass density of fluid is 1268.4 kg per cubic metre and kinematic viscosity is 6.30×10^{-4} square metre per second. Calculate the shear stress at the point. (*U.P.S.C., 1975*) [**Ans.** 0.2 N/m^2]
8. Find the kinematic viscosity of an oil having density 980 kg/m^2 when at a certain point in the oil, the shear stress is 0.25 N/m^2 and velocity gradient 0.3/s. $\left[\textbf{Ans. } 0.000849 \dfrac{\text{m}^2}{\text{sec}} \text{ or } 8.49 \text{ stokes}\right]$
9. Determine the specific gravity of a fluid having viscosity 0.07 poise and kinematic viscosity 0.042 stokes. [**Ans.** 1.667]
10. Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 2.0. [**Ans.** 11.99 poise]
11. If the velocity distribution of a fluid over a plate is given by $u = (3/4)\,y - y^2$, where u is the velocity in metre per second at a distance of y metres above the plate, determine the shear stress at $y = 0.15$ metre. Take dynamic viscosity of the fluid as 8.5×10^{-5} kg-sec/m^2. (*A.M.I.E., Winter 1974*) [**Ans.** 3.825×10^{-5} kgf/m^2]
12. An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of shaft is 0.5 m and it rotates at 200 r.p.m. Calculate the power lost in the oil for a sleeve length of 100 mm. The thickness of the oil film is 1.0 mm. [**Ans.** 2.15 kW]
13. The velocity distribution over a plate is given by $u = \dfrac{2}{3}\,y - y^2$ in which u is the velocity in m/sec at a distance of y m above the plate. Determine the shear stress at $y = 0$, 0.1 and 0.2 m. Take $\mu = 6$ poise. [**Ans.** 0.4, 0.028 and 0.159 N/m^2]
14. In question 13, find the distance in metres above the plate, at which the shear stress is zero. [**Ans.** 0.333 m]
15. The velocity profile of a viscous fluid over a plate is parabolic with vertex 20 cm from the plate, where the velocity is 120 cm/s. Calculate the velocity gradient and shear stress at distances of 0, 5 and 15 cm from the plate, given the viscosity of the fluid = 6 poise.[**Ans.** 12/s, 7.18 N/m^2; 9/s, 5.385 N/m^2 ; 3/s, 1.795 N/m^2]
16. The weight of a gas is given as 17.658 N/m^3 at 30°C and at an absolute pressure of 29.43 N/cm^2. Determine the gas constant and also the density of the gas. $\left[\textbf{Ans. } \dfrac{1.8 \text{ kg}}{\text{m}^3}, \dfrac{539.55 \text{ N-m}}{\text{kg}^\circ \text{K}}\right]$
17. A cylinder of 0.9 m^3 in volume contains air at 0°C and 39.24 N/cm^2 absolute pressure. The air is compressed to 0.45 m^3. Find (*i*) the pressure inside the cylinder assuming isothermal process, (*ii*) pressure and temperature assuming adiabatic process. Take $k = 1.4$ for air. [**Ans.** (*i*) 78.48 N/cm^2, (*ii*) 103.5 N/m^2, 140°C]
18. Calculate the pressure exerted by 4 kg mass of nitrogen gas at a temperature of 15°C if the volume is 0.35 m^3. Molecular weight of nitrogen is 28. [**Ans.** 97.8 N/cm^2]
19. The pressure of a liquid is increased from 60 N/cm^2 to 100 N/cm^2 and volume decreases by 0.2 per cent. Determine the bulk modulus of elasticity. [**Ans.** 2×10^4 N/cm^2]
20. Determine the bulk modulus of elasticity of a fluid which is compressed in a cylinder from a volume of 0.009 m^3 at 70 N/cm^2 pressure to a volume of 0.0085 m^3 at 270 N/cm^2 pressure. [**Ans.** 3.6×10^3 N/cm^2]
21. The surface tension of water in contact with air at 20°C is given as 0.0716 N/m. The pressure inside a droplet of water is to be 0.0147 N/cm^2 greater than the outside pressure, calculate the diameter of the droplet of water. [**Ans.** 1.94 mm]
22. Find the surface tension in a soap bubble of 30 mm diameter when the inside pressure is 1.962 N/m^2 above atmosphere. [**Ans.** 0.00735 N/m]
23. The surface tension of water in contact with air is given as 0.0725 N/m. The pressure outside the droplet of water of diameter 0.02 mm is atmospheric $\left(10.32 \dfrac{\text{N}}{\text{cm}^2}\right)$. Calculate the pressure within the droplet of water. [**Ans.** 11.77 N/cm^2]

24. Calculate the capillary rise in a glass tube of 3.0 mm diameter when immersed vertically in (*a*) water, and (*b*) mercury. Take surface tensions for mercury and water as 0.0725 N/m and 0.52 N/m respectively in contact with air. Specific gravity for mercury is given as 13.6. [**Ans.** 0.966 cm, 0.3275 cm]

25. The capillary rise in the glass tube used for measuring water level is not to exceed 0.5 mm. Determine its minimum size, given that surface tension for water in contact with air = 0.07112 N/m. [**Ans.** 5.8 cm]

26. (**SI Units**). One litre of crude oil weighs 9.6 N. Calculate its specific weight, density and specific gravity. (*Converted to SI units, A.M.I.E., Summer 1986*) [**Ans.** 9600 N/m^3; 979.6 kg/m^3 ; 0.9786]

27. (**SI Units**). A piston 796 mm diameter and 200 mm long works in a cylinder of 800 mm diameter. If the annular space is filled with a lubricating oil of viscosity 5 cp (centi-poise), calculate the speed of descent of the piston in vertical position. The weigh of the piston and axial load are 9.81 N. [**Ans.** 7.84 m/s]

28. (**SI Units**). Find the capillary rise of water in a tube 0.03 cm diameter. The surface tension of water is 0.0735 N/m. [**Ans.** 9.99 cm]

29. Calculate the specific weight, density and specific gravity of two litres of a liquid which weight 15 N. (*Delhi University, April 1992*) [**Ans.** 7500 N/m^3, 764.5 kg/m^3, 0.764]

30. A 150 mm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 151 mm. Both the cylinders are of 250 mm height. The space between the cylinders is filled with a liquid of viscosity 10 poise. Determine the torque required to rotate the inner cylinder at 100 r.p.m. (*Delhi University, April 1992*) [**Ans.** 13.87 Nm]

31. A shaft of diameter 120 mm is rotating inside a journal bearing of diameter 122 mm at a speed of 360 r.p.m. The space between the shaft and the bearing is filled with a lubricating oil of viscosity 6 poise. Find the power absorbed in oil if the length of bearing is 100 mm.(*Delhi University, May 1998*) [**Ans.** 115.73 W]

32. A shaft of diameter 100 mm is rotating inside a journal bearing of diameter 102 mm at a space of 360 r.p.m. The space between the shaft and bearing is filled with a lubricating oil of viscosity 5 poise. The length of the bearing is 200 mm. Find the power absorbed in the lubricating oil. (*Delhi University, June 1996*) [**Ans.** 111.58 W]

33. Assuming that the bulk modulus of elasticity of water is 2.07×10^6 kN/m^2 at standard atmospheric conditions, determine the increase of pressure necessary to produce 1% reduction in volume at the same temperature. (*Delhi University, June 1997*)

[**Hint.** $K = 2.07 \times 10^6$ kN/m^2 ; $\frac{-d\forall}{\forall} = \frac{1}{100} = 0.01$.

Increase in pressure (dp) $= K \times \left(\frac{-d\forall}{\forall}\right) = 2.07 \times 10^6 \times 0.01 = 2.07 \times 10^4$ kN/m^2. **Ans.**]

34. A square plate of size 1 m× 1 m and weighing 350 N slides down an inclined plane with a uniform velocity of 1.5 m/s. The inclined plane is laid on a slope of 5 vertical to 12 horizontal and has an oil film of 1 mm thickness. Calculate the dynamic viscosity of oil. [*J.N.T.U., Hyderabad, S 2002*]

[**Hint.** $A = 1 \times 1 = 1$ m^2, W = 350 N, $u = 1.5$ m/s, $\tan\theta = \frac{5}{12} = \frac{BC}{AB}$

Component of weight along the plane = W × sin θ

where $\sin\theta = \frac{BC}{AC} = \frac{5}{13}$ $\left(\because AC = \sqrt{AB^2 + BC^2} = \sqrt{12^2 + 5^2} = 13\right)$

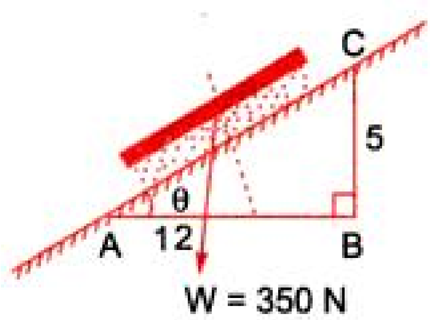


Fig. 1.15

∴ $F = W\sin\theta = 350 \times \frac{5}{13} = 134.615$

Now $\tau = \mu \frac{du}{dy}$, where $du = u - 0 = u = 1.5$ m/s and

$dy = 1$ mm $= 1 \times 10^{-3}$ m

or $\frac{F}{A} = \mu \frac{du}{dy}$, $\therefore \mu = \frac{F}{A} \times \frac{dy}{du} = \frac{134.615}{1} \times \frac{1\times10^{-3}}{1.5} = 0.0897 \frac{Ns}{m^2} = 0.897$ poise **Ans.**]

2

CHAPTER

Pressure and its Measurement

2.1 FLUID PRESSURE AT A POINT

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

The units of pressure are : (*i*) kgf/m^2 and kgf/cm^2 in MKS units, (*ii*) Newton/m^2 or N/m^2 and N/mm^2 in SI units. N/m^2 is known as Pascal and is represented by Pa. Other commonly used units of pressure are :

$$\text{kPa} = \text{kilo pascal} = 1000 \text{ N/m}^2$$
$$\text{bar} = 100 \text{ kPa} = 10^5 \text{ N/m}^2.$$

2.2 PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, dx, dy and ds.

Consider an **arbitrary** fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x,

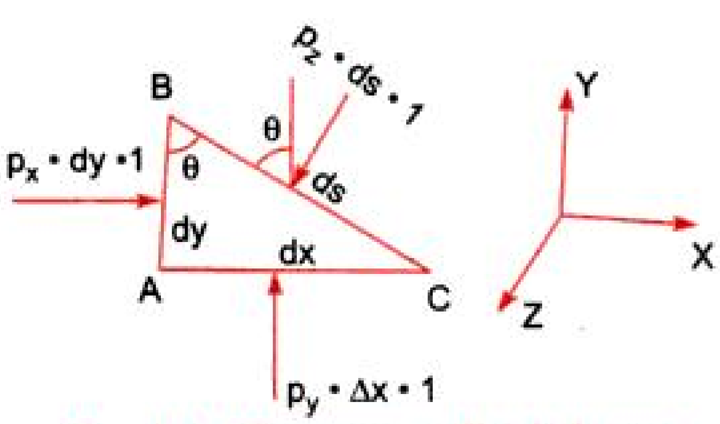


Fig. 2.1 *Forces on a fluid element.*

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces.
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $=$ (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1) \sin (90° - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos \theta = 0.$$

But from Fig. 2.1, $ds \cos \theta = AB = dy$

$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z \qquad \text{...(2.1)}$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos (90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin \theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin \theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

or $$p_y = p_z \qquad \text{...(2.2)}$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad \text{...(2.3)}$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

▶ 2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

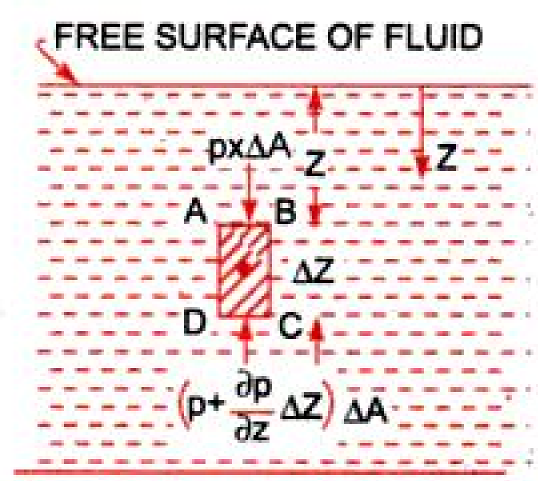


Fig. 2.2 Forces on a fluid element.

The forces acting on the fluid element are :

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.

2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.

3. Weight of fluid element = Density × g × Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or
$$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or
$$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or
$$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$$\therefore \quad \frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because p \times g = w) \qquad \text{...(2.4)}$$

where w = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g Z$$

or
$$p = \rho g Z \qquad \text{...(2.5)}$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have $Z = \frac{p}{\rho \times g}$ ...(2.6)

Here Z is called **pressure head.**

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = .00159 \text{ m}^2$

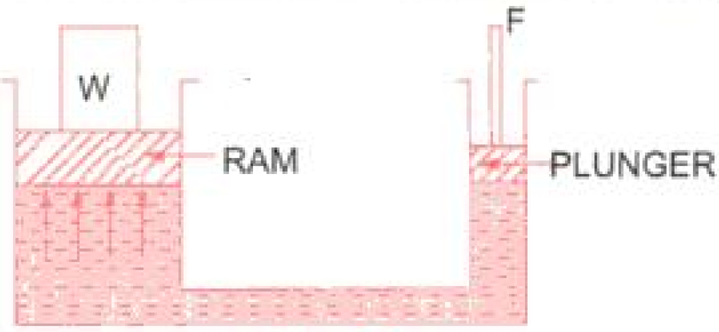


Fig. 2.3

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} =$ **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20 \text{ cm} = 0.2 \text{ m}$

$\therefore$ Area of ram, $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3 \text{ cm} = 0.03 \text{ m}$

$\therefore$ Area of plunger, $a = \dfrac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000 \text{ N}.$

See Fig. 2.3.

Pressure intensity developed due to plunger $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \dfrac{F}{a}$

$\therefore$ Force acting on ram = Pressure intensity $\times$ Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \textbf{675.2 N. Ans.}$$

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000 \text{ kg/m}^3$.*

Solution. Given :

Height of liquid column, $Z = 0.3 \text{ m}.$

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2. \text{ Ans.}}$$

(*b*) For oil of sp. gr. 0.8,

From equation (1.1*A*), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}. \text{ Ans.}}$$

(*c*) For mercury, sp. gr. = 13.6

From equation (1.1A) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, $\rho_s = \text{Specific gravity of mercury} \times \text{Density of water}$

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$\therefore$ $p = \rho_s \times g \times Z$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}. \text{ Ans}}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}.$

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \mathbf{4 \text{ m of water. Ans.}}$

(*b*) For oil, sp. gr. = 0.9

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$\therefore$ $Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \mathbf{4.44 \text{ m of oil. Ans.}}$

Problem 2.5 *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Solution. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40$ m

Density of oil, ρ_0 = Sp. gr. of oil × Density of water = 0.9 × 1000 = 900 kg/m^3

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \ \frac{\text{N}}{\text{m}^2}$

∴ Corresponding height of water $= \dfrac{p}{\text{Density of water} \times g}$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \textbf{36 m of water. Ans.}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, ρ_2 = Sp. gr. of oil × Density of water = 0.9 × 1000 = 900 kg/m^3

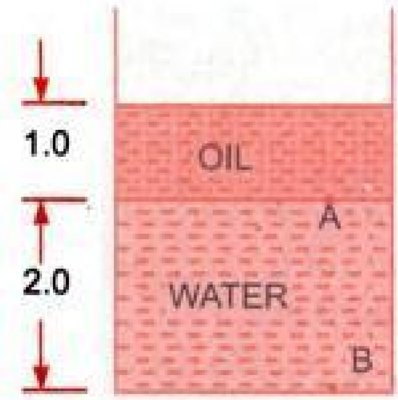


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \ \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \textbf{0.8829 N/cm}^2\textbf{. Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$
$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \textbf{2.8449 N/cm}^2\textbf{. Ans.}$$

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack ate 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m^3.

Solution. Given :

Dia. of small piston, $d = 3$ cm

∴ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(*a*) **When the pistons are at the same level**

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

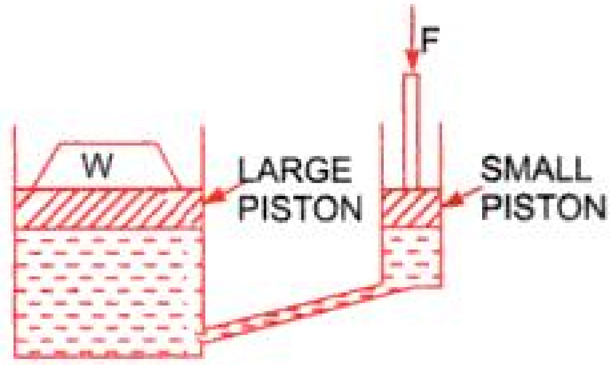


Fig. 2.5

(*b*) **When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section $A - A$

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

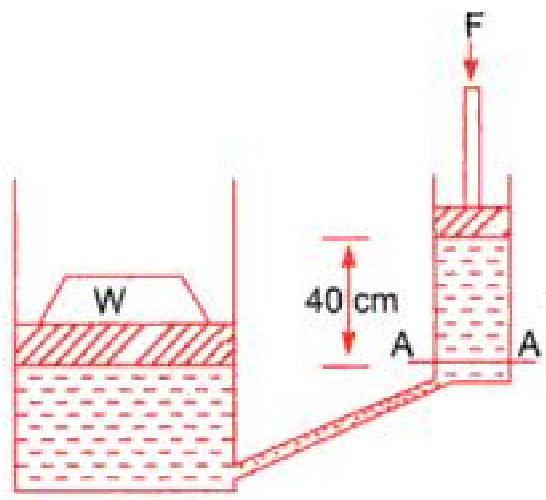


Fig. 2.6

$\therefore$ Pressure intensity at section

$$A - A = \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston $= 11.71$ N/cm^2

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▸ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. **Vacuum pressure** is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.

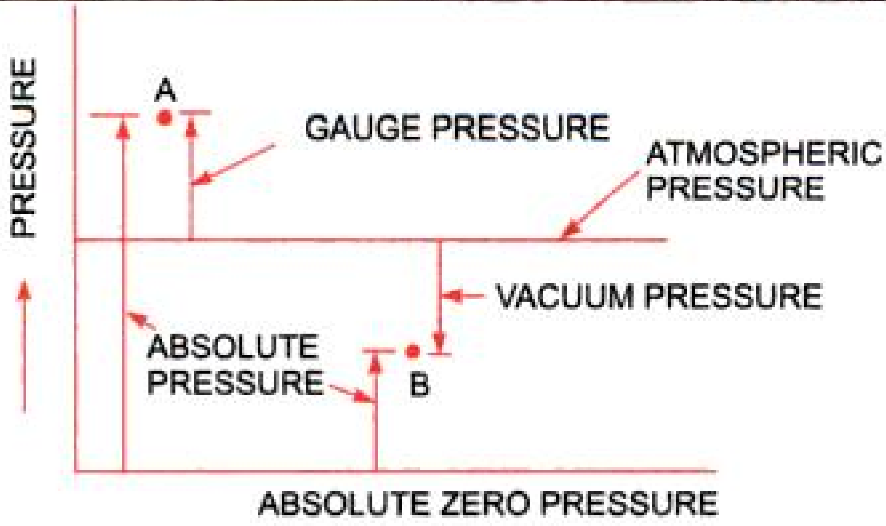


Fig. 2.7 *Relationship between pressures.*

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m² or 10.13 N/cm² in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm².

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m³ if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m³.*

(A.M.I.E., Summer 1986)

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m³

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6×1000 kg/m³

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, p = **45028 N/m². Ans**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m². Ans.**

2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers
2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

▶ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

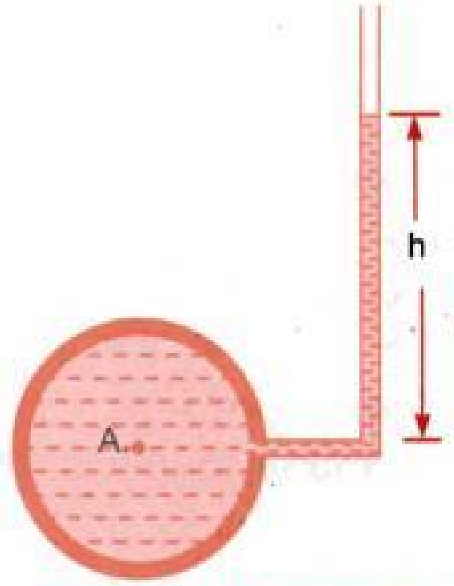


Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

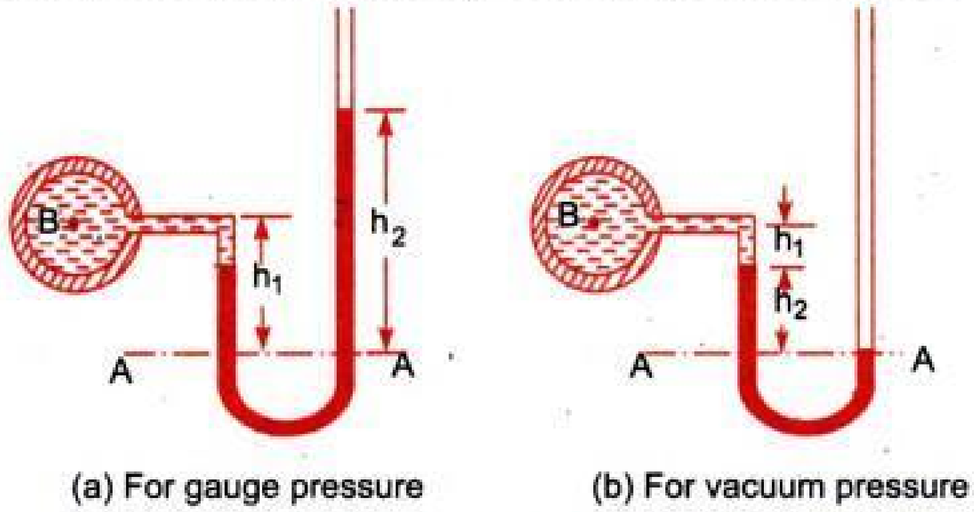


Fig. 2.9 *U-tube Manometer.*

(*a*) For Gauge Pressure. Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1). \quad \text{...(2.7)}$$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A = 0

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1). \quad \text{...(2.8)}$$

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

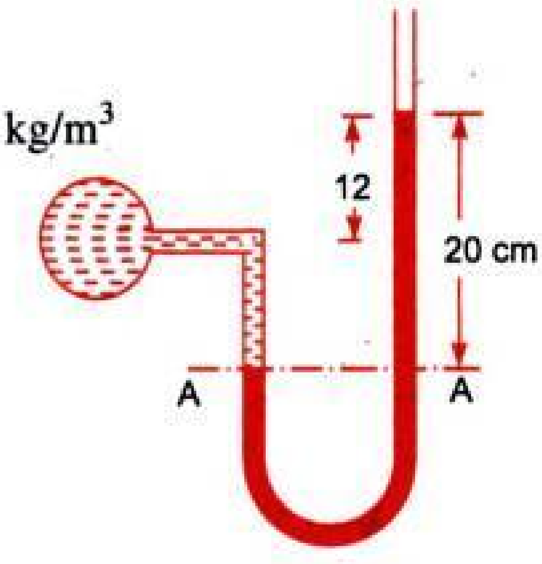


Fig. 2.10

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000$ kg/m^3

Difference of mercury level $h_2 = 20$ cm $= 0.2$ m

Height of fluid from *A-A*, $h_1 = 20 - 12 = 8$ cm $= 0.08$ m

Let p = Pressure of fluid in pipe

Equating the pressure above *A-A*, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or
$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597\ N/cm^2}. \textbf{ Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

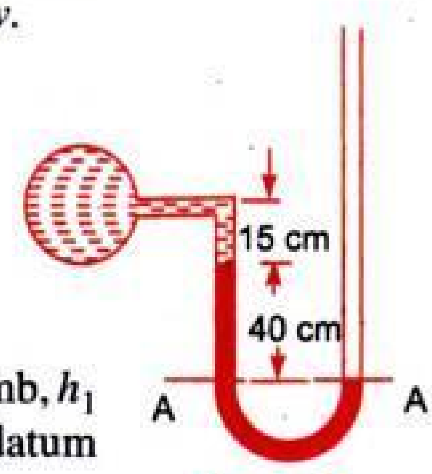


Fig. 2.11

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Difference of mercury level, $h_2 = 40$ cm $= 0.4$ m. Height of liquid in left limb, $h_1 = 15$ cm $= 0.15$ m. Let the pressure in pipe $= p$. Equating pressure above datum line *A-A*, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -[\rho_2 g h_2 + \rho_1 g h_1]$$
$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$
$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.* (A.M.I.E., Winter 1989)

Solution. Given :

Difference of mercury = 10 cm = 0.1 m

The arrangement is shown in Fig. 2.11 (*a*)

Let p_A = (pressure of water in pipe line (*i.e.*, at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*

= Pressure at *A* + Pressure due to 10 cm (or 0.1 m) of water

$$= p_A + \rho \times g \times h$$

where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1$ m

$$= p_A + 1000 \times 9.81 \times 0.1$$
$$= p_A + 981 \text{ N/m}^2 \quad \text{...(i)}$$

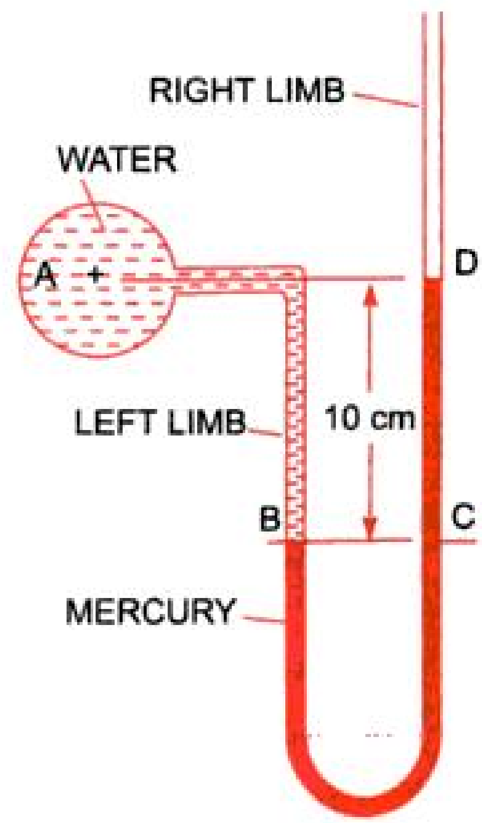


Fig. 2.11 (*a*)

Pressure at *C* = Pressure at *D* + Pressure due to 10 cm of mercury

$$= 0 + \rho_0 \times g \times h_0$$

where ρ_0 for mercury = $13.6 \times 1000 \text{ kg/m}^3$

and h_0 = 10 cm = 0.1 m

$$\therefore \quad \text{Pressure at } C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$$
$$= 13341.6 \text{ N} \quad \text{...(ii)}$$

But pressure at *B* is equal to pressure at *C*. Hence equating the equations (*i*) and (*ii*), we get

$$p_A + 981 = 13341.6$$
$$\therefore \quad p_A = 13341.6 - 981$$
$$= \mathbf{12360.6 \frac{N}{m^2}}. \textbf{ Ans.}$$

IInd Part

Given, $p_A = 9810 \text{ N/m}^2$

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (*b*). In this case the pressure at *A* is 9810 N/m² which is less than the 12360.6 N/m². Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points *B*, *C* and *D* show the initial conditions whereas points *B**, *C** and *D** show the final conditions.

The pressure at B^* = Pressure at C^*

or Pressure at A + Pressure due to $(10 - x)$ cm of water

= Pressure at D^* + Pressure due to $(10 - 2x)$ cm of mercury

or $$p_A + \rho_1 \times g \times h_1 = p_D^* + \rho_2 \times g \times h_2$$

or $$1910 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right)$$

$$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

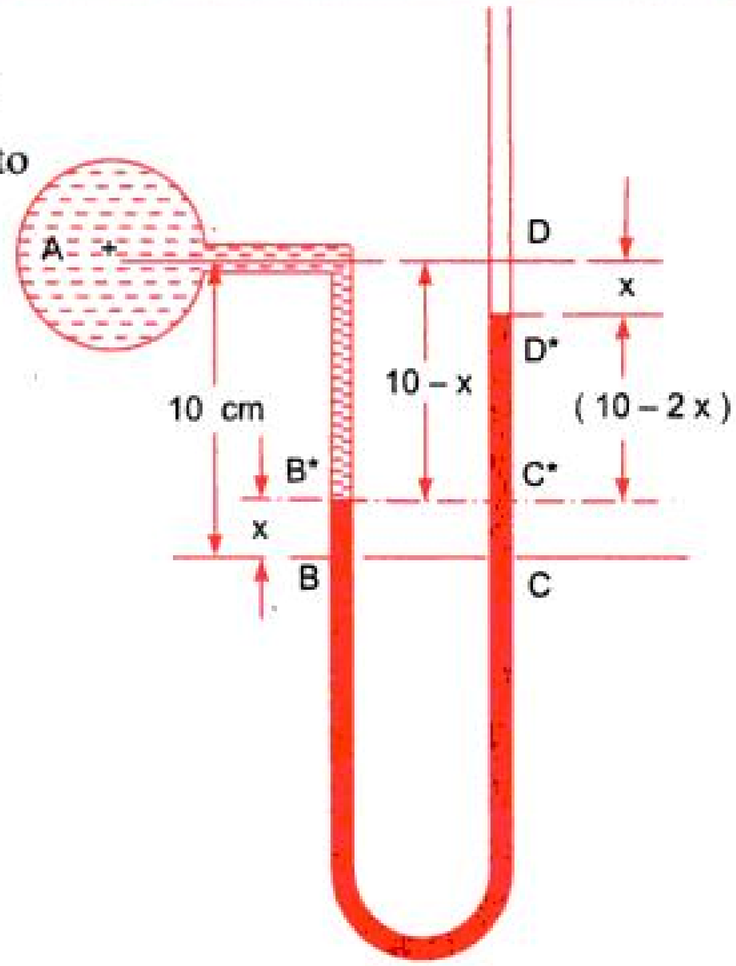


Fig. 2.11 (*b*)

Dividing by 9.81, we get

or $$1000 + 100 - 10x = 1360 - 272x$$

or $$272x - 10x = 1360 - 1100$$

or $$262x = 260$$

$\therefore$ $$x = \frac{260}{262} = 0.992 \text{ cm}$$

$\therefore$ New difference of mercury $= 10 - 2x$ cm $= 10 - 2 \times 0.992$

$= \mathbf{8.016}$ **cm. Ans.**

Problem 2.12 *Fig. 2.12 shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.* (A.M.I.E., Winter 1975)

Solution. Vessel is empty. Given :

Difference of mercury level $h_2 = 20$ cm

Let h_1 = Height of water above X-X

Sp. gr. of mercury, $S_2 = 13.6$

Sp. gr. of water, $S_1 = 1.0$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Density of water, $\rho_1 = 1000$

Equating the pressure above datum line X-X, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

or $$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water.}$$

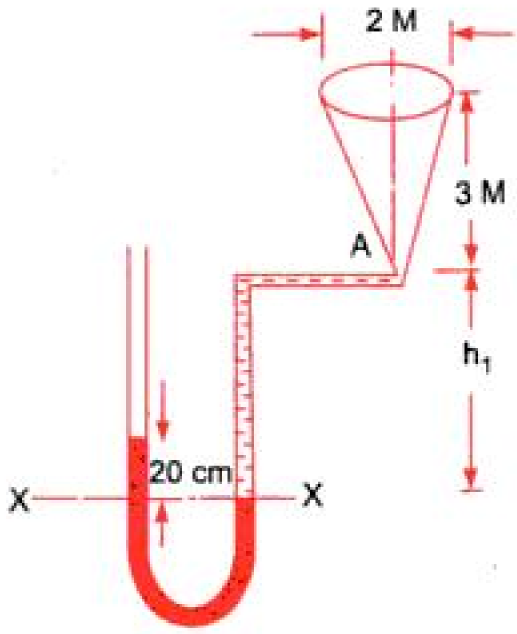


Fig. 2.12

Vessel is full of water. When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in Fig. 2.13. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.

Pressure in left limb = Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100)$$

$$= 1000 \times 9.81 \times (3 + h_1 + y/100)$$

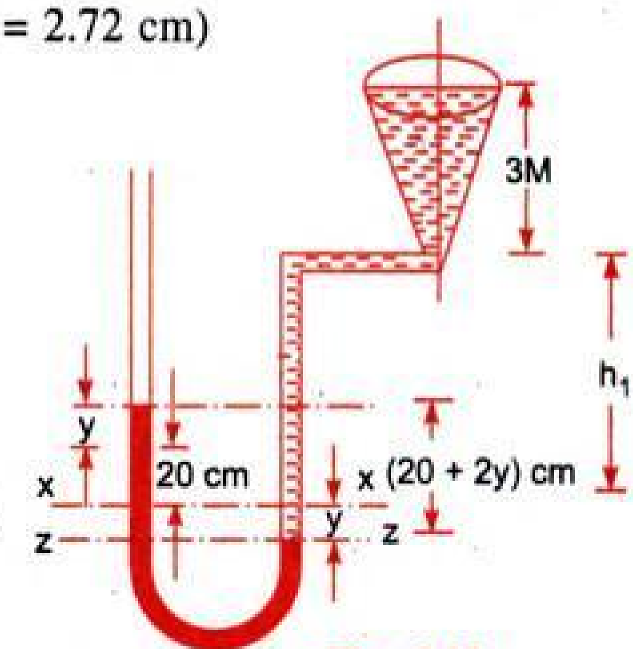


Fig. 2.13

or $\quad 13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \quad (\because \; h_1 = 2.72 \text{ cm})$

or $\quad 2.72 + 27.2y/100 = 3 + 2.72 + y/100$

or $\quad (27.2y - y)/100 = 3.0$

or $\quad 26.2y = 3 \times 100 = 300$

$$\therefore \quad y = \frac{300}{26.2} = 11.45 \text{ cm}$$

The difference of mercury level in two limbs

$= (20 + 2y)$ cm of mercury

$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90$ cm of mercury

$\therefore$ Reading of manometer = **42.90 cm. Ans.**

Problem 2.13 *A pressure gauge consists of two cylindrical bulbs B and C each of 10 sq. cm cross-sectional area, which are connected by a U-tube with vertical limbs each of 0.25 sq. cm cross-sectional area. A red liquid of specific gravity 0.9 is filled into C and clear water is filled into B, the surface of separation being in the limb attached to C. Find the displacement of the surface of separation when the pressure on the surface in C is greater than that in B by an amount equal to 1 cm head of water.* (A.M.I.E., Summer, 1978)

Solution. Given :

Area of each bulb B and C, $\quad A = 10 \text{ cm}^2$

Area of each vertical limb, $\quad a = 0.25 \text{ cm}^2$

Sp. gr. of red liquid $\quad = 0.9 \quad \therefore$ Its density $= 900 \text{ kg/m}^3$

Let $\quad X\text{-}X$ = Initial separation level

h_C = Height of red liquid above X-X

h_B = Height of water above X-X

Pressure above X-X in the left limb $= 1000 \times 9.81 \times h_B$

Pressure above X-X in the right limb $= 900 \times 9.81 \times h_C$

Equating the two pressure, we get

$$1000 \times 9.81 \times h_B = 900 \times 9.81 \times h_C$$

$$\therefore \quad h_B = 0.9 \; h_C \quad \ldots(i)$$

When the pressure head over the surface in C is increased by 1 cm of water, let the separation level falls by an amount equal to Z. Then Y-Y becomes the final separation level.

Now fall in surface level of C multiplied by cross-sectional area of bulb C must be equal to the fall in separation level multiplied by cross-sectional area of limb.

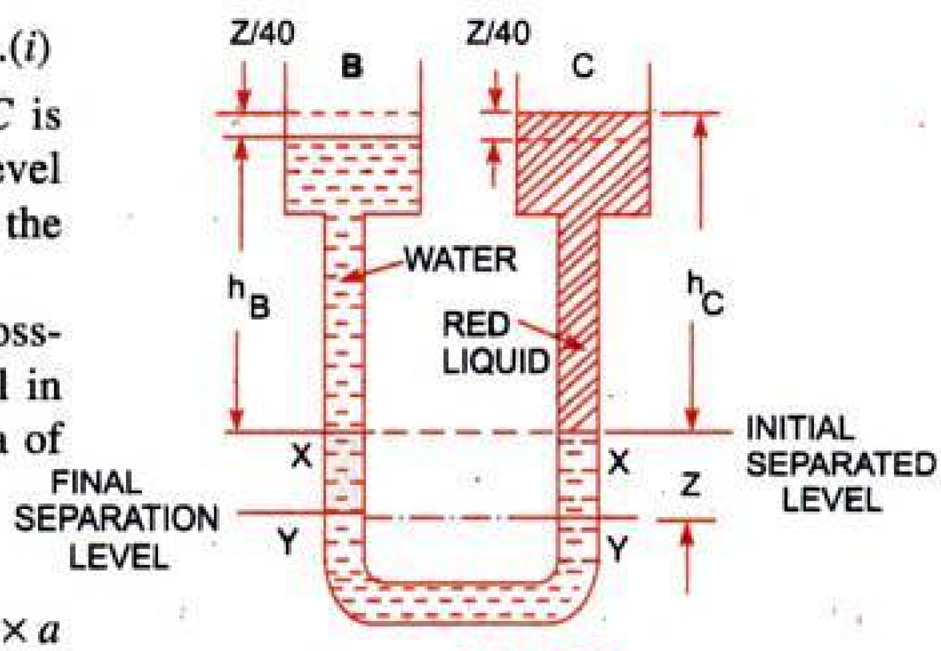


Fig. 2.14

$\therefore$ Fall in surface level of C

$$= \frac{\text{Fall in separation level} \times a}{A}$$

$$= \frac{Z \times a}{A} = \frac{Z \times 0.25}{10} = \frac{Z}{40}.$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water = $\rho g h$ = 1000 × 9.81 × 0.01= 98.1 N/m^2

Consider final separation level Y-Y

Pressure above Y-Y in the left limb = $1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right)$

Pressure above Y-Y in the right limb = $900 \times 9.81\left(Z + h_C - \frac{Z}{40}\right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right) = \left(Z + h_C - \frac{Z}{40}\right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40}\right) = 900 \left(Z + h_C - \frac{Z}{40}\right) + 10$$

Dividing by 1000, we get $Z + h_B + \frac{Z}{40} = 0.9 \left(Z + h_C - \frac{Z}{40}\right) + 0.01$

But from equation (i), $\quad h_B = 0.9\ h_C$

$$\therefore \quad Z + 0.9\ h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\ h_C + 0.01$$

or

$$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or

$$Z\left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z\left(\frac{41 - 35.1}{40}\right) = .01$$

$$\therefore \quad Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

2.6.3 Single Column Manometer. Single column manomter is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

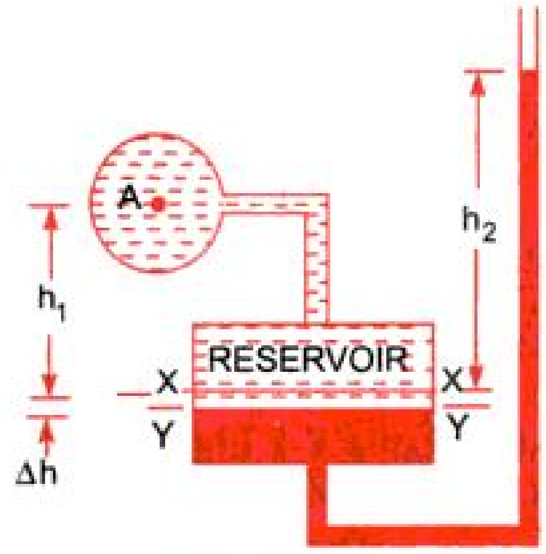


Fig. 2.15 *Vertical single column manometer.*

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \quad \ldots(i)$$

Now consider the datum line Y-Y as shown in Fig. 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (i),

$$\Delta h = \frac{a \times h_2}{A}$$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g \quad \ldots(2.9)$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g$ ...(2.10)

From equation (2.10), it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

2. Inclined Single Column Manometer

Fig. 2.16 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

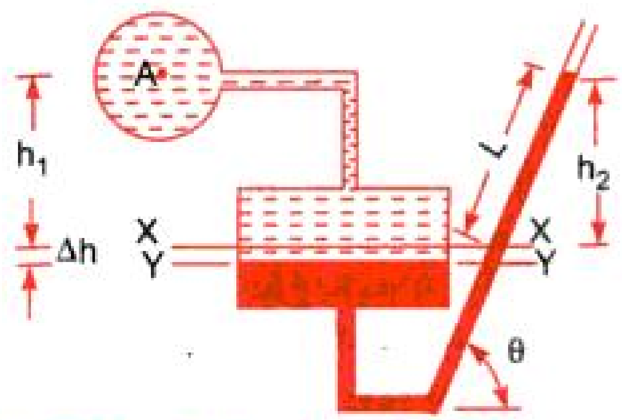


Fig. 2.16 *Inclined single column manometer.*

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-$X = L \times \sin\theta$

From equation (2.10), the pressure at A is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of h_2, we get

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g. \quad \text{...(2.11)}$$

Problem 2.14 *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let p_A = Pressure in pipe

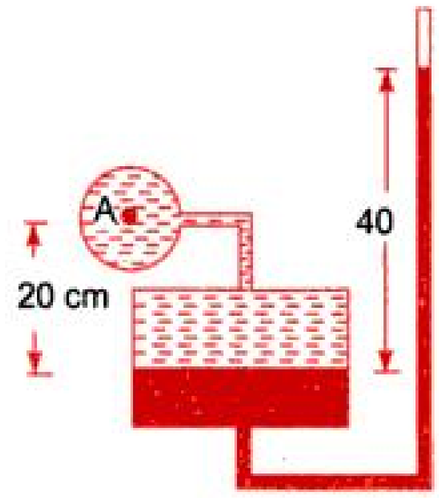


Fig. 2.17

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}[133416 - 8829] + 53366.4 - 1765.8$$

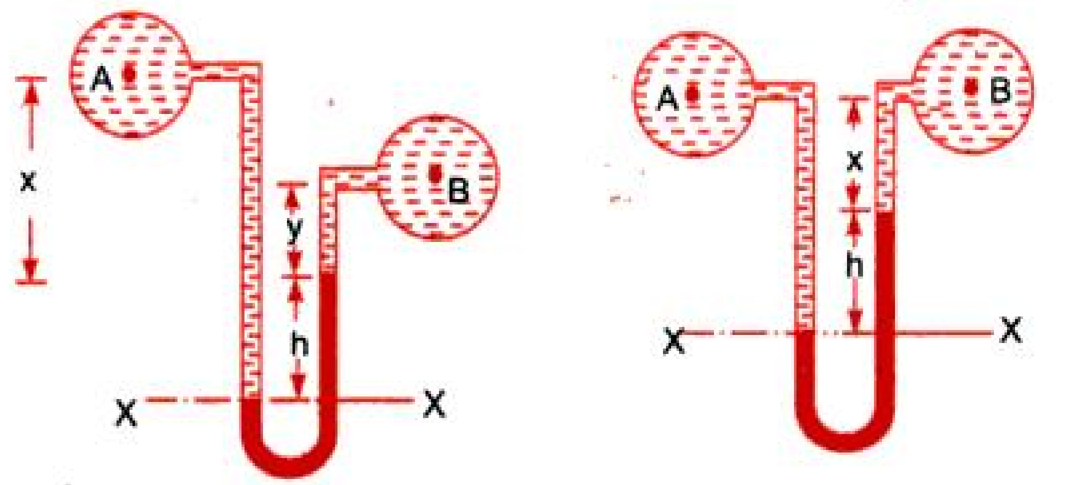
$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm². Ans.**

▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1 U-tube Differential Manometer. Fig. 2.18 shows the differential manometers of U-tube type.



(a)Two pipes at different levels (b) A and B are at the same level

Fig. 2.18 *U-tube differential manometers.*

Fig. 2.18 (*a*). Let the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \qquad \text{...(2.12)}$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

Fig. 2.18 (*b*). A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \qquad \text{...(2.13)}$$

Problem 2.15 *A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.*

Solution. Given :

Sp. gr. of oil, $S_1 = 0.9$ $\therefore$ Density, $\rho_1 = 0.9 \times 1000 = 900$ kg/m^3

Difference in mercury level, $h = 15$ cm $= 0.15$ m

Sp. gr. of mercury, $S_g = 13.6$ $\therefore$ Density, $\rho_g = 13.6 \times 1000$ kg/m^3

The difference of pressure is given by equation (2.13)

or $$p_A - p_B = g \times h(\rho_g - \rho_1)$$
$$= 9.81 \times 0.15\,(13600 - 900) = \mathbf{18688\ N/m^2.\ Ans.}$$

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential manometer.*

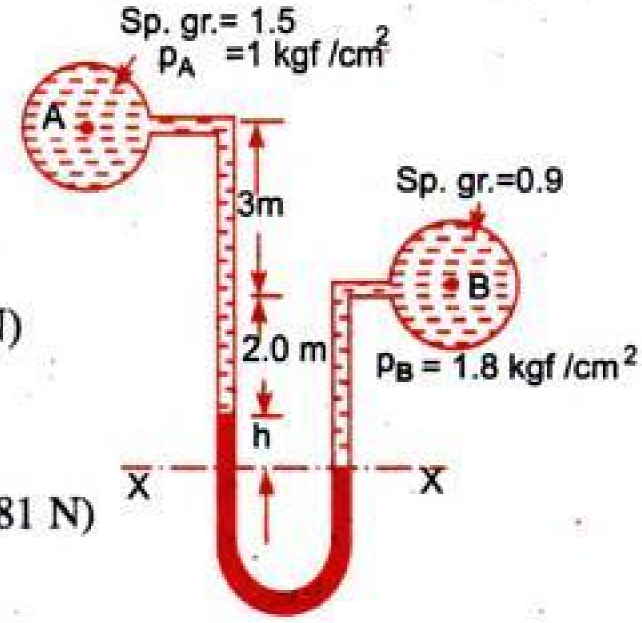


Fig. 2.19

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm^2 $= 1 \times 10^4$ kgf/m^2
$= 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm^2
$= 1.8 \times 10^4$ kgf/m^2
$= 1.8 \times 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m^3

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1\ cm.\ Ans.}$$

Problem 2.17 *A differential manometer is connected at the two points A and B as shown in Fig. 2.20. At B air pressure is 9.81 N/cm^2 (abs), find the absolute pressure at A.*

Solution. Given :

Air pressure at $B = 9.81$ N/cm^2

or $p_B = 9.81 \times 10^4$ N/m^2

Density of oil $= 0.9 \times 1000 = 900$ kg/m^3

Density of mercury $= 13.6 \times 1000$ kg/m^3

Let the pressure at A is p_A

Taking datum line at X-X

Pressure above X-X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B$$
$$= 5886 + 98100 = 103986$$

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$
$$= 13341.6 + 1765.8 + p_A$$

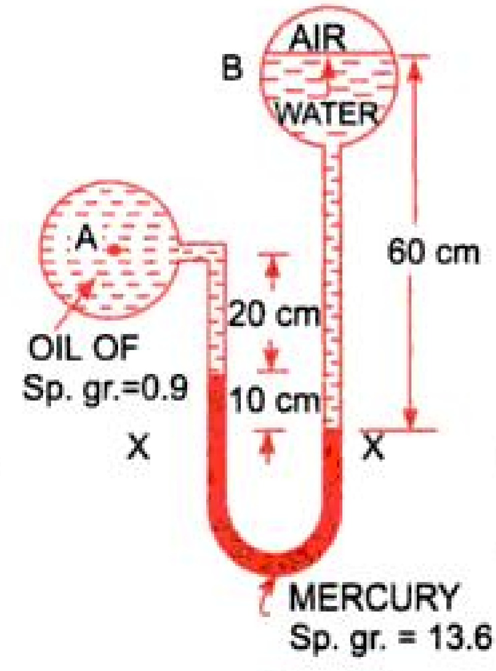


Fig. 2.20

Equating the two pressure head

$$103986 = 13341.6 + 1765.8 + p_A$$

$\therefore$ $$p_A = 103986 - 15107.4 = 88876.8$$

$\therefore$ $$p_A = 88876.8 \text{ N/m}^2 = \frac{88876.8 \text{N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}.$$

$\therefore$ Absolute pressure at A = **8.887 N/cm². Ans.**

2.7.2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

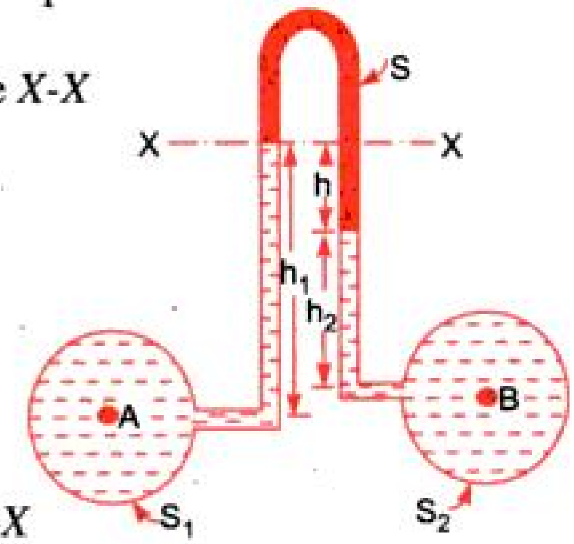


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or $$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18 *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Solution. Given :

Pressure head at $A = \frac{p_A}{\rho g} = 2$ m of water

$\therefore$ $$p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$$

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb $= p_A - \rho_1 \times g \times h_1$

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $$p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$$

or $$p_B = \mathbf{1.8599 \text{ N/cm}^2}. \textbf{ Ans.}$$

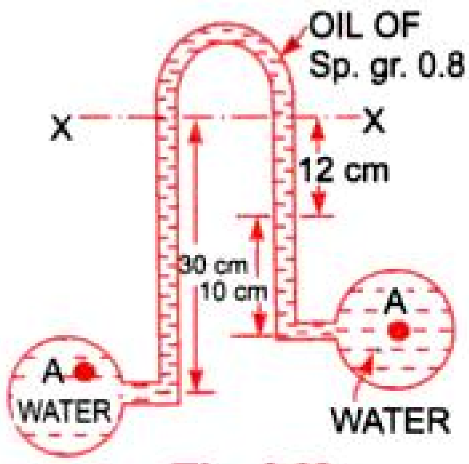


Fig. 2.22

Problem 2.19 *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

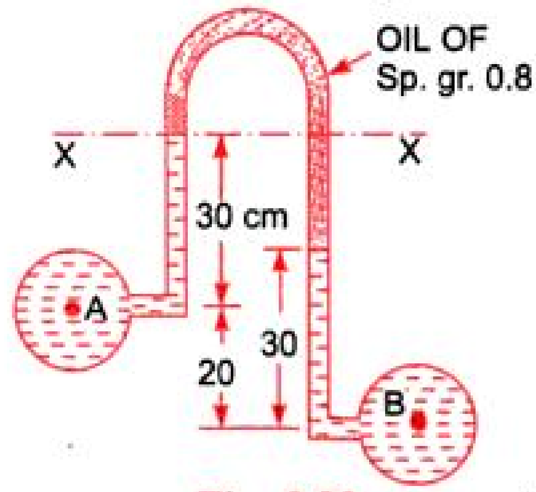


Fig. 2.23

Solution. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$$= (30 + 20) - 30 = 20 \text{ cm}$$

Taking datum line at X-X

Pressure in the left limb below X-X

$$= p_A - 1000 \times 9.81 \times 0$$

$$= p_A - 2943$$

Pressure in the right limb below X-X

$$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$$

$$= p_B - 2943 - 1569.6 = p_B - 4512.6$$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$$\therefore \quad p_B - p_A = 4512.6 - 2943 = \mathbf{1569.6 \text{ N/m}^2}. \textbf{ Ans.}$$

Problem 2.20 *Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in Fig. 2.24 below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.*

(A.M.I.E., Winter 1985)

Solution. Given :

Fig. 2.24 shows the arrangement. Taking X-X as datum line.

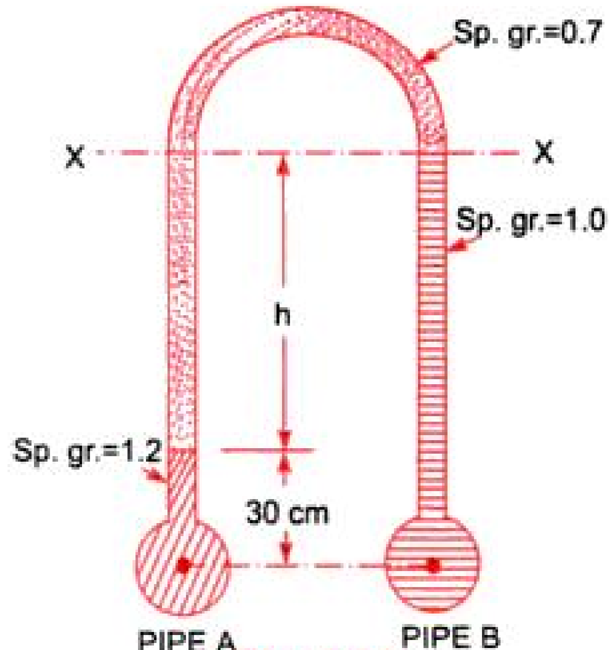


Fig. 2.24

Let p_A = Pressure at A

p_B = Pressure at B

Density of liquid in pipe A = Sp. gr. × 1000

$= 1.2 \times 1000$

$= 1200 \text{ kg/m}^2$

Density of liquid in pipe B $= 1 \times 1000 = 1000 \text{ kg/m}^3$

Density of oil $= 0.7 \times 1000 = 700 \text{ kg/m}^3$

Now pressure below X-X in the left limb

$$= p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times (h + 0.3)$$

Equating the two pressure, we get

$$p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = p_B - 1000 \times 9.81\ (h + 0.3)$$

But $\quad p_A = p_B$ (given)

$\therefore \quad - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = - 1000 \times 9.81\ (h + 0.3)$

Dividing by 1000×9.81

$$- 1.2 \times 0.3 - 0.7h = - (h + 0.3)$$

or $\quad 0.3 \times 1.2 + 0.7h = h + 0.3$ or $0.36 - 0.3 = h - 0.7h = 0.3h$

$$\therefore \quad h = \frac{0.36 - 0.30}{0.30} = \frac{0.06}{0.30} \text{ m}$$

$$= \frac{1}{5} \text{ m} = \frac{1}{5} \times 100 = \mathbf{20\ cm.\ Ans.}$$

Problem 2.21 *An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30 cm. When an oil of specific gravity 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from tlhe respective centre lines of the pipes) are found to be same and equal to 35 cm. Determine the difference of pressure between the pipes.*

(A.M.I.E., Summer 1990)

Solution. Given :

Specific gravity of measuring liquid = 0.8

The arrangement is shown in Fig. 2.24 (a).

Let $\quad p_A$ = pressure at A

$\quad p_B$ = pressure at B.

The points C and D lie on the same horizontal line.

Hence pressure at C should be equal to pressure at D.

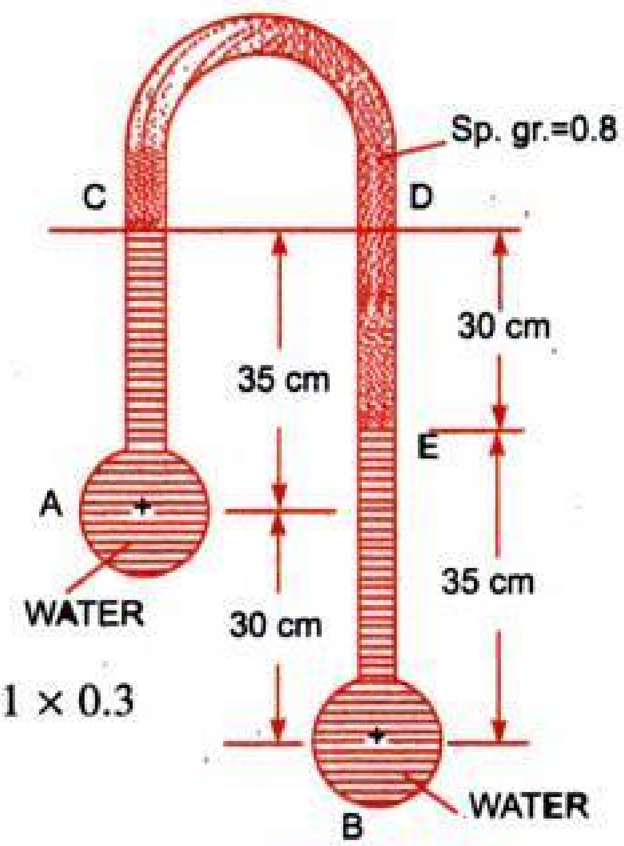


Fig. 2.24 (*a*)

But pressure at $C \quad = p_A - \rho g\ h$

$\quad = p_A - 1000 \times 9.81 \times (0.35)$

And pressure at $D \quad = p_B - \rho_1 g h_1 - \rho_2 g h_2$

$\quad = p_B - 1000 \times 9.81 \times (0.35) - 800 \times 9.81 \times 0.3$

But pressure at C = pressure at D

$\therefore \quad p_A - 1000 \times 9.81 \times .35$

$\quad = p_B - 1000 \times 9.81 \times 0.35 - 800 \times 9.81 \times 0.3$

or $\quad 800 \times 9.81 \times 0.3 = p_B - p_A$

or $$p_B - p_A = 800 \times 9.81 \times 0.3 = \mathbf{2354.4\ \frac{N}{m^2}.\ Ans.}$$

▶ 2.8 PRESSURE AT A POINT IN COMPRESSIBLE FLUID

For compressible fluids, density (ρ) changes with the change of pressure and temperature. Such problems are encountered in aeronautics, oceanography and meteorology where we are concerned with atmospheric* air where density, pressure and temperature changes with elevation. Thus for fluids with variable density, equation (2.4) cannot be integrated, unless the relationship between p and ρ is known. For gases the equation of state is

$$\frac{p}{\rho} = RT \qquad ...(2.15)$$

or
$$\rho = \frac{p}{RT}$$

Now equation (2.4) is
$$\frac{dp}{dZ} = w = \rho g = \frac{p}{RT} \times g$$

$$\therefore \quad \frac{dp}{p} = \frac{g}{RT} dZ \qquad ...(2.16)$$

In equation (2.4), Z is measured vertically downward. But if Z is measured vertically up, then $\frac{dp}{dZ} = -\rho g$ and hence equation (2.16) becomes

$$\frac{dp}{p} = \frac{-g}{RT} dZ \qquad ...(2.17)$$

2.8.1 Isothermal Process. **Case I.** If temperature T is constant which is true for **isothermal process**, equation (2.17) can be integrated as

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT} dz = -\frac{g}{RT}\int_{Z_0}^{Z} dz$$

or
$$\log \frac{p}{p_0} = \frac{-g}{RT} [Z - Z_0]$$

where p_0 is the pressure where height is Z_0. If the datum line is taken at Z_0, then $Z_0 = 0$ and p_0 becomes the pressure at datum line.

$$\therefore \quad \log \frac{p}{p_0} = \frac{-g}{RT} Z$$

$$\frac{p}{p_0} = e^{-gZ/RT}$$

or pressure at a height Z is given by $p = p_0 e^{-gZ/RT}$ ...(2.18)

2.8.2 Adiabatic Process. If temperature T is not constant but the process follows adiabatic law then the relation between pressure and density is given by

$$\frac{p}{\rho^k} = \text{Constant} = C \qquad ...(i)$$

* The standard atmospheric pressure, temperature and density referred to STP at the sea-level are : Pressure = 101.325 kN/m^2 ; Temperature = 15°C and Density = 1.225 kg/m^3.

where k is ratio of specific constant.

$$\therefore \qquad \rho^k = \frac{p}{C}$$

$$\text{or} \qquad \rho = \left(\frac{p}{C}\right) \qquad \ldots(ii)$$

Then equation (2.4) for Z measured vertically up becomes,

$$\frac{dp}{dZ} = -\rho g = -\left(\frac{p}{C}\right)^{1/k} g$$

$$\text{or} \qquad \frac{dp}{\left(\frac{p}{C}\right)^{1/k}} = -gdZ \text{ or } C^{1/k}\frac{dp}{\frac{1}{p^k}} = -gdZ$$

Integrating, we get $\int_{p_0}^{p} C^{1/k}\, p^{-1/k}\, dp = \int_{Z_0}^{Z} -gdZ$

$$\text{or} \qquad C^{1/k}\left[\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

$$\text{or} \qquad \left[\frac{C^{1/k}\, p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z} \qquad [C \text{ is a constant, can be taken inside}]$$

But from equation (i), $\qquad C^{1/k} = \left(\frac{p}{\rho^k}\right)^{1/k} = \frac{p^{1/k}}{\rho}$

Substituting this value of $C^{1/k}$ above, we get

$$\left[\frac{p^{1/k}}{\rho}\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g[Z - Z_0]$$

$$\text{or} \qquad \left[\frac{p^{\frac{1-1+k}{k}}}{\rho^{\frac{k-1}{k}}}\right]_{p_0}^{p} = -g[Z - Z_0] \text{ or } \left[\frac{k}{k-1}\frac{p}{\rho}\right]_{p_0}^{p} = -g[Z - Z_0]$$

$$\text{or} \qquad \frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -g[Z - Z_0]$$

If datum line is taken at Z_0, where pressure, temperature and density are p_0, T_0 and ρ_0, then $Z_0 = 0$.

$$\therefore \qquad \frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -gZ \quad \text{or} \quad \frac{p}{\rho} - \frac{p_0}{\rho_0} = -gZ\,\frac{(k-1)}{k}$$

$$\text{or} \qquad \frac{p}{\rho} = \frac{p_0}{\rho_0} - gZ\,\frac{(k-1)}{k} = \frac{p_0}{\rho_0}\left[1 - \frac{k-1}{k}\,gZ\,\frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{\rho} \times \frac{\rho_0}{p_0} = \left[1 + \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right] \quad ...(iii)$$

But from equation (*i*), $$\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k} \quad \text{or} \quad \left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p} \text{ or } \frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/k}$$

Substituting the value of $\frac{\rho_0}{\rho}$ in equation (*iii*), we get

$$\frac{p}{p_0} \times \left(\frac{p_0}{p}\right)^{1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{p_0} \times \left(\frac{p}{p_0}\right)^{-1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\left(\frac{p}{p_0}\right)^{1-\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

$\therefore$ $$\frac{p}{p_0} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}}$$

$\therefore$ Pressure at a height Z from ground level is given by

$$p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} \quad ...(2.19)$$

in equation (2.19), p_0 = pressure at ground level, where $Z_0 = 0$

ρ_0 = density of air at ground level

Equation of state is $$\frac{p_0}{\rho_0} = RT_0 \text{ or } \frac{\rho_0}{p_0} = \frac{1}{RT_0}$$

Substituting the values of $\frac{\rho_0}{p_0}$ in equation (2.19), we get

$$p = p_0 \left[1 - \frac{k-1}{k} \frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \quad ...(2.20)$$

2.8.3 Temperature at any Point in Compressible Fluid. For the adiabatic process, the temperature at any height in air is calculated as :

Equation of state at ground level and at a height Z from ground level is written as

$$\frac{p_0}{\rho_0} = RT_0 \text{ and } \frac{p}{\rho} = RT$$

Dividing these equations, we get

$$\left(\frac{p_0}{\rho_0}\right) + \frac{p}{\rho} = \frac{RT_0}{RT} = \frac{T_0}{T} \quad \text{or} \quad \frac{p_0}{\rho_0} \times \frac{\rho}{p} = \frac{T_0}{T}$$

or $$\frac{T}{T_0} = \frac{\rho_0}{p_0} \times \frac{p}{\rho} = \frac{p}{p_0} \times \frac{\rho_0}{\rho} \quad ...(i)$$

But $\frac{p}{p_0}$ from equation (2.20) is given by

$$\frac{p}{p_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

Also for adiabatic process $\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k}$ or $\left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p}$

or
$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{-\frac{1}{k}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\left(\frac{k}{k-1}\right)\times\left(-\frac{1}{k}\right)} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

Substituting the values of $\frac{p}{p_0}$ and $\frac{\rho_0}{\rho}$ in equation (*i*), we get

$$\frac{T}{T_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \times \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}-\frac{1}{k-1}} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

$$\therefore \qquad T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right] \qquad \text{...(2.21)}$$

2.8.4 Temperature Lapse-Rate (L). It is defined as the rate at which the temperature changes with elevation. To obtain an expression for the temperature lapse-rate, the temperature given by equation (2.21) is differentiated with respect to Z as

$$\frac{dT}{dZ} = \frac{d}{dZ}\left[T_0\left(1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right)\right]$$

where T_0, K, g and R are constant

$$\therefore \qquad \frac{dT}{dZ} = -\frac{k-1}{k} \times \frac{g}{RT_0} \times T_0 = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

The temperature lapse-rate is denoted by L and hence

$$L = \frac{dT}{dZ} = \frac{-g}{R}\left(\frac{k-1}{k}\right) \qquad \text{...(2.22)}$$

In equation (2.22), if (*i*) $k = 1$ which means isothermal process, $\frac{dT}{dZ} = 0$, which means temperature is constant with height.

(*ii*) If $k > 1$, the lapse-rate is negative which means temperature decreases with the increase of height.

In atmosphere, the value of k varies with height and hence the value of temperature lapse-rate also varies. From the sea-level upto an elevation of about 11000 m (or 11 km), the temperature of air decreases uniformly at the rate of 0.0065°C/m. from 11000 m to 32000 m, the temperature remains constant at – 56.5°C and hence in this range lapse-rate is zero. Temperature rises again after 32000 m in air.

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*i*) **Pressure by hydrostatic law.** For hydrostatic law, ρ is assumed constant and hence p is given by equation $\frac{dp}{dZ} = -\rho g$

Integrating, we get $$\int_{p_0}^{p} dp = \int -\rho g \, dZ = -\rho g \int_{Z_0}^{Z} dZ$$

or $$p - p_0 = -\rho g \, [Z - Z_0]$$

For datum line at sea-level, $Z_0 = 0$

$$\therefore \quad p - p_0 = -\rho g Z \quad \text{or} \quad p = p_0 - \rho g Z$$

$$= 10.143 \times 10^4 - 1.208 \times 9.81 \times 2500 \; [\because \rho = \rho_0 = 1.208]$$

$$= 101430 - 29626 = 71804 \frac{\text{N}}{\text{m}^2} \text{ or } \frac{71804}{10^4} \text{ N/cm}^2$$

$$= \mathbf{7.18 \text{ N/cm}^2. \text{ Ans.}}$$

(*ii*) **Pressure by Isothermal Law.** Pressure at any height Z by isothermal law is given by equation (2.18) as

$$p = p_0 \, e^{-gZ/RT}$$

$$= 10.143 \times 10^4 \, e^{-\frac{gZ \times \rho_0}{p_0}} \qquad \left[\because \frac{p_0}{\rho_0} = RT \text{ and } \rho_0 g = w_0\right]$$

$$= 10.143 \times 10^4 \, e^{-\frac{Z\rho_0 \times g}{p_0}}$$

$$= 10.143 \times 10^4 \, e^{(-2500 \times 1.208 \times 9.81)/10.143 \times 10^4}$$

$$= 101430 \times e^{-.292} = 101430 \times \frac{1}{1.3391} = 75743 \text{ N/m}^2$$

$$= \frac{75743}{10^4} \text{ N/cm}^2 = \mathbf{7.574 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.23 *The barometric pressure at sea level is 760 mm of mercury while that on a mountain top is 735 mm, if the density of air is assumed constant at 1.2 kg/m³, what is the elevation of the mountain top.* (A.M.I.E., Summer, 1988)

Solution. Given :

Pressure* at sea, $p_0 = 760$ mm of Hg

$$= \frac{760}{1000} \times 13.6 \times 1000 \times 9.81 \text{ N/m}^2 = 101396 \text{ N/m}^2$$

---

* Here pressure head (Z) is given as 760 mm of Hg. Hence $(p/\rho g) = 760$ mm of Hg. The density (ρ) for mercury $= 13.6 \times 1000 \text{ kg/m}^3$. Hence pressure ($p$) will be equal to $\rho \times g \times Z$ *i.e.*, $13.6 \times 1000 \times 9.81 \times \frac{760}{1000} \text{ N/m}^2$.

Pressure at mountain, $p = 735$ mm of Hg

$$= \frac{735}{1000} \times 13.6 \times 1000 \times 9.81 = 98060 \text{ N/m}^2$$

Density of air, $\rho = 1.2 \text{ kg/m}^3$

Let h = Height of the mountain from sea-level.

We know that as the elevation above the sea-level increases, the atmospheric pressure decreases. Here the density of air is given constant, hence the pressure at any height 'h' above the sea-level is given by the equation,

$$p = p_0 - \rho \times g \times h$$

or
$$h = \frac{p_0 - p}{\rho \times g} = \frac{101396 - 98060}{1.2 \times 9.81} = \mathbf{283.33 \text{ m. Ans.}}$$

Problem 2.24 *Calculate the pressure at a height of 7500 m above sea level if the atmospheric pressure is 10.143 N/cm² and temperature is 15°C at the sea-level, assuming (i) air is incompressible, (ii) pressure variation follows isothermal law, and (iii) pressure variation follows adiabatic law. Take the density of air at the sea-level as equal to 1.285 kg/m³. Neglect variation of g with altitude.*

Solution. Given :

Height above sea-level, $Z = 7500$ m

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Temperature at sea-level, $t_0 = 15°\text{C}$

$\therefore$ $T_0 = 273 + 15 = 288°\text{K}$

Density of air, $\rho = \rho_0 = 1.285 \text{ kg/m}^3$

(*i*) Pressure when air is incompressible :

$$\frac{dp}{dZ} = -\rho g$$

$\therefore$
$$\int_{p_0}^{p} dp = -\int_{Z_0}^{Z} \rho g dz \quad \text{or} \quad p - p_0 = -\rho g[Z - Z_0]$$

or
$$p = p_0 - \rho g Z \qquad \{\because \; Z_0 = \text{datum line} = 0\}$$
$$= 10.143 \times 10^4 - 1.285 \times 9.81 \times 7500$$
$$= 101430 - 94543 = 6887 \text{ N/m}^2 = \mathbf{0.688 \frac{N}{cm^2}. \text{ Ans.}}$$

(*ii*) Pressure variation follows isothermal law :

Using equation (2.18), we have $p = p_0 e^{-gZ/RT}$

$$= p_0 \, e^{-gZ\rho_0/p_0} \qquad \left\{\because \; \frac{p_0}{\rho_0} = RT \;\therefore\; \frac{\rho_0}{p_0} = \frac{1}{RT}\right\}$$
$$= 101430 \, e^{-gZ\rho_0/p_0} = 101430 \, e^{-7500 \times 1.285 \times 9.81/101430}$$
$$= 101430 \, e^{-.9320} = 101430 \times .39376$$
$$= \mathbf{39939 \text{ N/m}^2 \text{ or } 3.993 \text{ N/cm}^2. \text{ Ans.}}$$

(*iii*) Pressure variation follows adiabatic law : [$k = 1.4$]

Using equation (2.19), we have
$$p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{k/(k-1)}, \text{ where } \rho_0 = 1.285 \text{ kg/m}^3$$

$$\therefore \quad p = 101430\left[1-\frac{(1.4-1.0)}{1.4}\times 9.81\times\frac{(7500\times 1.285)}{101430}\right]^{\frac{1.4}{1.4-1.0}}$$

$$= 101430\,[1-.2662]^{1.4/.4} = 101430 \times (.7337)^{3.5}$$

$$= \mathbf{34310\ N/m^2\ or\ 3.431\ \frac{N}{cm^2}.\ Ans.}$$

Problem 2.25 *Calculate the pressure and density of air at a height of 4000 m from sea-level where pressure and temperature of the air are 10.143 N/cm² and 15°C respectively. The temperature lapse rate is given as 0.0065°C/m. Take density of air at sea-level equal to 1.285 kg/m³.*

Solution. Given :

Height, $Z = 4000$ m

Pressure at sea-level, $p_0 = 10.143\ \text{N/cm}^2 = 10.143 \times 10^4 = 101430\ \dfrac{\text{N}}{\text{m}^2}$

Temperature at sea-level, $t_0 = 15°\text{C}$

$\therefore$ $T_0 = 273 + 15 = 288°\text{K}$

Temperature lapse-rate, $L = \dfrac{dT}{dZ} = -\ 0.0065°\text{K/m}$

$\rho_0 = 1.285\ \text{kg/m}^3$

Using equation (2.22), we have $L = \dfrac{dT}{dZ} = -\dfrac{g}{R}\left(\dfrac{k-1}{k}\right)$

or $$-\ 0.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right),\ \text{where } R = \frac{p_0}{\rho_0 T_0} = \frac{101430}{1.285\times 288} = 274.09$$

$$\therefore \quad -\ 0.0065 = \frac{-9.81}{274.09}\times\left(\frac{k-1}{k}\right)$$

$$\therefore \quad \frac{k-1}{k} = \frac{0.0065\times 274.09}{9.81} = 0.1815$$

$$\therefore \quad k[1-.1815] = 1$$

$$\therefore \quad k = \frac{1}{1-.1815} = \frac{1}{.8184} = 1.222$$

This means that the value of power index $k = 1.222$.

(*i*) **Pressure** at 4000 m height is given by equation (2.19) as

$$p = p_0\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}},\ \text{where } k = 1.222 \text{ and } \rho_0 = 1.285$$

$$\therefore \quad p = 101430\left[1-\left(\frac{1.222-1.0}{1.222}\right)\times 9.81\times\frac{4000\times 1.285}{101430}\right]^{\frac{1.222}{1.222-1.0}}$$

$$= 101430\,[1-0.09]^{5.50} = 101430 \times .595$$

$$= 60350\ \text{N/m}^2 = \mathbf{6.035\ \frac{N}{cm^2}.\ Ans.}$$

(*ii*) **Density.** Using equation of state, we get

$$\frac{p}{\rho} = RT$$

where p = Pressure at 4000 m height
ρ = Density at 4000 m height
T = Temperature at 4000 m height

Now T is calculated from temperature lapse-rate as

$$t \text{ at } 4000 \text{ m} = t_0 + \frac{dT}{dZ} \times 4000 = 15 - .0065 \times 4000 = 15 - 26 = -11°\text{C}$$

$$\therefore \qquad T = 273 + t = 273 - 11 = 262°\text{K}$$

$\therefore$ Density is given by $\rho = \dfrac{p}{RT} = \dfrac{60350}{274.09 \times 262}$ kg/m^3 = **0.84 kg/m^3. Ans.**

Problem 2.26 *An aeroplane is flying at an altitude of 5000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065°K/m. Neglect variation of g with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C and density of air as 1.285 kg/cm^3.*

Solution. Given :

Height, $Z = 5000$ m

Lapse-rate, $L = \dfrac{dT}{dZ} = -.0065°$K/m

Pressure at ground level, $p_0 = 10.143 \times 10^4$ N/m^2

$t_0 = 15°$C

$\therefore$ $T_0 = 273 + 15 = 288°$K

Density, $\rho_0 = 1.285$ kg/m^3

$\therefore$ Temperature at 5000 m height $= T_0 + \dfrac{dT}{dZ} \times \text{Height} = 288 - .0065 \times 5000$

$$= 288 - 32.5 = 255.5°\text{K}$$

First find the value of power index k as

From equation (2.22), we have $L = \dfrac{dT}{dZ} = -\dfrac{g}{R}\left(\dfrac{k-1}{k}\right)$

or

$$-.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right)$$

where $R = \dfrac{p_0}{\rho_0 T_0} = \dfrac{101430}{1.285 \times 288} = 274.09$

$$\therefore \qquad -.0065 = -\frac{9.81}{274.09}\left(\frac{k-1}{k}\right)$$

$$\therefore \qquad k = 1.222$$

The pressure is given by equation (2.19) as

$$p = p_0\left[1 - \frac{k-1}{k}\, gZ\, \frac{\rho_0}{p_0}\right]^{\left(\frac{k}{k-1}\right)}$$

$$= 101430 \left[1 - \left(\frac{1.222 - 1.0}{1.222}\right) \times 9.81 \times \frac{5000 \times 1.285}{101430}\right]^{\frac{1.222}{1.222 - 1.0}}$$

$$= 101430 \left[1 - \frac{.222}{1.222} \times 9.81 \times \frac{5000 \times 1.285}{101430}\right]^{\frac{1.222}{.222}}$$

$= 101430\ [1 - 0.11288]^{5.50} = 101430 \times 0.5175 = 52490\ \text{N/m}^3$

= 5.249 N/cm². Ans.

HIGHLIGHTS

1. The pressure at any point in a fluid is defined as the force per unit area.
2. The Pascal's law states that intensity of pressure for a fluid at rest is equal in all directions.
3. Pressure variation at a point in a fluid at rest is given by the hydrostatic law which states that the rate of increase of pressure in the vertically downward direction is equal the specific weight of the fluid,
   $\frac{dp}{dZ} = w = \rho \times g.$
4. The pressure at any point in a incompressible fluid (liquid) is equal to the product of density of fluid at that point, acceleration due to gravity and vertical height from free surface of fluid, $p = \rho \times g \times Z.$
5. Absolute pressure is the pressure in which absolute vacuum pressure is taken as datum while gauge pressure is the pressure in which the atmospheric pressure is taken as datum,
   $$p_{abs.} = p_{atm} + p_{gauge}.$$
6. Manometer is a device used for measuring pressure at a point in a fluid.
7. Manometers are classified as (*a*) Simple manometers and (*b*) Differential manometers.
8. Simple manometers are used for measuring pressure at a point while differential manometers are used for measuring the difference of pressures between the two points in a pipe, or two different pipes.
9. A single column manometer (or micrometer) is used for measuring small pressures, where accuracy is required.
10. The pressure at a point in static compressible fluid is obtained by combining two equations, *i.e.*, equation of state for a gas and equation given by hydrostatic law.
11. The pressure at a height Z in a static compressible fluid (gas) under going isothermal compression $\left(\frac{p}{\rho} = \text{const.}\right)$
    $$p = p_0\, e^{-gZ/RT}$$
    where p_0 = Absolute pressure at sea-level or at ground level
    Z = Height from sea or ground level
    R = Gas constant
    T = Absolute temperature.
12. The pressure and temperature at a height Z in a static compressible fluid (gas) undergoing adiabatic compression (p/ρ^k = const.)
    $$p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} = p_0 \left[1 - \frac{k-1}{k} \frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

and temperature, $$T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0, T_0 are pressure and temperature at sea-level $k = 1.4$ for air.

13. The rate at which the temperature changes with elevation is known as Temperature Lapse-Rate. It is given by

$$L = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

if (*i*) $k = 1$, temperature is zero.

(*ii*) $k > 1$, temperature decreases with the increase of height.

EXERCISE 2

(A) THEORETICAL PROBLEMS

1. Define pressure. Obtain an expression for the pressure intensity at a point in a fluid.
2. State and prove the Pascal's law.
3. What do you understand by Hydrostatic Law ?
4. Differentiate between : (*i*) Absolute and gauge pressure, (*ii*) Simple manometer and differential manometer, and (*iii*) Piezometer and pressure gauges.
5. What do you mean by vacuum pressure ?
6. What is a manometer ? How are they classified ?
7. What do you mean by single column manometers ? How are they used for the measurement of pressure ?
8. What is the difference between U-tube differential manometers and inverted U-tube differential manometers ? Where are they used ?
9. Distinguish between manometers and mechanical gauges. What are the different types of mechanical pressure gauges ?
10. Derive an expression for the pressure at a height Z from sea-level for a static air when the compression of the air is assumed isothermal. The pressure and temperature at sea-levels are p_0 and T_0 respectively.
11. Prove that the pressure and temperature for an adiabatic process at a height Z from sea-level for a static air are :

$$p_0 = p_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \text{ and } T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0 and T_0 are the pressure and temperature at sea-level.

12. What do you understand by the term, 'Temperature Lapse-Rate' ? Obtain an expression for the temperature Lapse-Rate.
13. What is hydrostatic pressure distribution ? Give one example where pressure distribution is non-hydrostatic. *(A.M.I.E., Winter 1990)*
14. Explain briefly the working principle of Bourdon Pressure Gauge with a neat sketch. *(J.N.T.U., Hyderabad, S 2002)*

(B) NUMERICAL PROBLEMS

1. A hydraulic press has a ram of 30 cm diameter and a plunger of 5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N. [**Ans.** 14.4 kN]
2. A hydraulic press has a ram of 20 cm diameter and a plunger of 4 cm diameter. It is used for lifting a weight of 20 kN. Find the force required at the plunger. [**Ans.** 800 N]

3. Calculate the pressure due to a column of 0.4 m of (*a*) water, (*b*) an oil of sp. gr. 0.9, and (*c*) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000 \frac{kg}{m^3}$. [**Ans.** (*a*) 0.3924 N/cm^2, (*b*) 0.353 N/cm^2, (*c*) 5.33 N/cm^2]
4. The pressure intensity at a point in a fluid is given 4.9 N/cm^2. Find the corresponding height of fluid when it is : (*a*) water, and (*b*) an oil of sp. gr. 0.8. [**Ans.** (*a*) 5 m of water, (*b*) 6.25 m of oil]
5. An oil of sp. gr. 0.8 is contained in a vessel. At a point the height of oil is 20 m. Find the corresponding height of water at that point. [**Ans.** 16 m]
6. An open tank contains water upto a depth of 1.5 m and above it an oil of sp. gr. 0.8 for a depth of 2 m. Find the pressure intensity : (*i*) at the interface of the two liquids, and (*ii*) at the bottom of the tank. [**Ans.** (*i*) 1.57 N/cm^2, (*ii*) 3.04 N/cm^2]
7. The diameters of a small piston and a large piston of a hydraulic jack are 2 cm and 10 cm respectively. A force of 60 N is applied on the small piston. Find the load lifted by the large piston, when : (*a*) the pistons are at the same level, and (*b*) small piston is 20 cm above the large piston. The density of the liquid in the jack is given as 1000 $\frac{kg}{m^2}$. [**Ans.** (*a*) 1500 N, (*b*) 1520.5 N]
8. Determine the gauge and absolute pressure at a point which is 2.0 m below the free surface of water. Take atmospheric pressure as 10.1043 N/cm^2. [**Ans.** 1.962 N/cm^2 (gauge), 12.066 N/cm^2 (abs.)]
9. A simple manometer is used to measure the pressure of oil (sp. gr. = 0.8) flowing in a pipe line. Its right limb is open to the atmosphere and left limb is connected to the pipe. The centre of the pipe is 9 cm below the level of mercury (sp. gr. 13.6) in the right limb. If the difference of mercury level in the two limbs is 15 cm, determine the absolute pressure of the oil in the pipe in N/cm^2. *(A.M.I.E., Winter, 1977)* [**Ans.** 12.058 N/cm^2]
10. A simple manometer (U-tube) containing mercury is connected to a pipe in which an oil of sp. gr. 0.8 is flowing. The pressure in the pipe is vacuum. The other end of the manometer is open to the atmosphere. Find the vacuum, pressure in pipe, if the difference of mercury level in the two limbs is 20 cm and height of oil in the left limb from the centre of the pipe is 15 cm below. [**Ans.** – 27.86 N/cm^2]
11. A single column vertical manometer (*i.e.*, micrometer) is connected to a pipe containing oil of sp. gr. 0.9. The area of the reservoir is 80 times the area of the manometer tube. The reservoir contains mercury of sp. gr. 13.6. The level of mercury in the reservoir is at a height of 30 cm below the centre of the pipe and difference of mercury levels in the reservoir and right limb is 50 cm. Find the pressure in the pipe. [**Ans.** 6.474 N/cm^2]
12. A pipe contains an oil of sp. gr. 0.8. A differential manometer connected at the two points A and B of the pipe shows a difference in mercury level as 20 cm. Find the difference of pressure at the two points. [**Ans.** 25113.6 N/m^2]
13. A U-tube differential manometer connects two pressure pipes *A* and *B*. Pipe *A* contains carbon tetrachloride having a specific gravity 1.594 under a pressure of 11.772 N/cm^2 and pipe *B* contains oil of sp. gr. 0.8 under a pressure of 11.772 N/cm^2. The pipe *A* lies 2.5 m above pipe *B*. Find the difference of pressure measured by mercury as fluid filling U-tube. *(A.M.I.E. December, 1974)* [**Ans.** 31.36 cm of mercury]
14. A differential manometer is connected at the two points *A* and *B* as shown in Fig. 2.25. At *B* air pressure is 7.848 N/cm^2 (abs.), find the absolute pressure at *A*. [**Ans.** 6.91 N/cm^2]

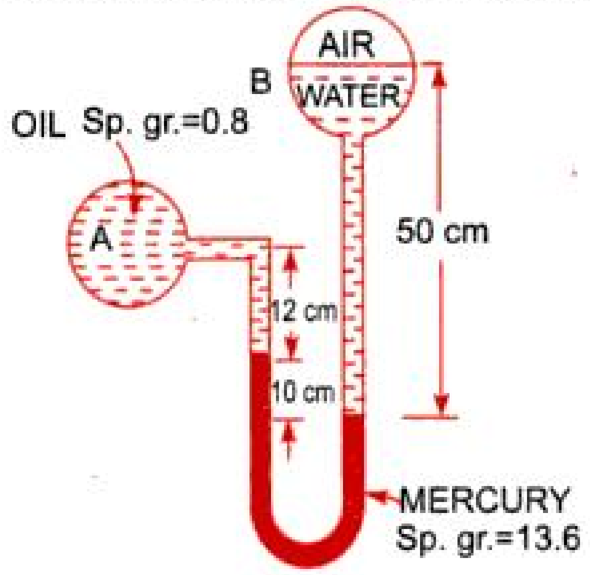


Fig. 2.25

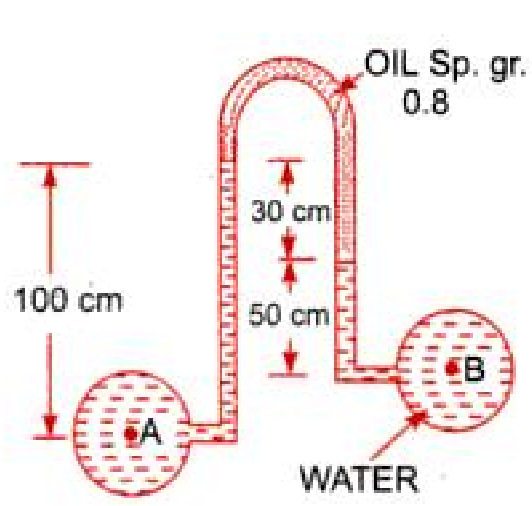


Fig. 2.26

15. An inverted differential manometer containing an oil of sp. gr. 0.9 is connected to find the difference of pressures at two points of a pipe containing water. If the manometer reading is 40 cm, find the difference of pressures. [**Ans.** 392.4 N/m^2]

16. In above Fig. 2.26 shows an inverted differential manometer connected to two pipes *A* and *B* containing water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the difference of pressure head between *A* and *B*. [**Ans.** 0.26 m of water]

17. If the atmospheric pressure at sea-level is 10.143 N/cm^2, determine the pressure at a height of 2000 m assuming that the pressure variation follows : (*i*) Hydrostatic law, and (*ii*) Isothermal law. The density of air is given as 1.208 kg/m^3. [**Ans.** (*i*) 7.77 N/cm^2, (*ii*) 8.03 N/cm^3]

18. Calculate the pressure at a height of 8000 m above sea-level if the atmospheric pressure is 101.3 kN/m^2 and temperature is 15°C at the sea-level assuming (*i*) air is incompressible, (*ii*) pressure variation follows adiabatic law, and (*iii*) pressure variation follows isothermal law. Take the density of air at the sea-level as equal to 1.285 kg/m^3. Neglect variation of *g* with altitude.
[**Ans.** (*i*) 607.5 N/m^2, (*ii*) 31.5 kN/m^2 (*iii*) 37.45 kN/m^2]

19. Calculate the pressure and density of air at a height of 3000 m above sea-level where pressure and temperature of the air are 10.143 N/cm^2 and 15°C respectively. The temperature lapse-rate is given as 0.0065° K/m. Take density of air at sea-level equal to 1.285 kg/m^3. [**Ans.** 6.896 N/cm^2, 0.937 kg/m^3]

20. An aeroplane is flying at an altitude of 4000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065° K/m. Neglect variation of *g* with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C respectively. The density of air at ground level is given as 1.285 kg/m^3. [**Ans.** 6.038 N/cm^2]

21. The atmosphere pressure at the sea-level is 101.3 kN/m^2 and the temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming (*i*) air is in compressible, (*ii*) isothermal variation of pressure and density, and (*iii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of '*g*' with altitude.
[**Ans.** (*i*) 501.3 N/m^2, (*ii*) 37.45 kN/m^2, (*iii*) 31.5 kN/m^2]

22. An oil of sp. gr. is 0.8 under a pressure of 137.2 kN/m^2
   (*i*) What is the pressure head expressed in metre of water ?
   (*ii*) What is the pressure head expressed in metre of oil ? [**Ans.** (*i*) 14 m, (*ii*) 17.5 m]

23. The atmospheric pressure at the sea-level is 101.3 kN/m^2 and temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming : (*i*) isothermal variation of pressure and density, and (*ii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of '*g*' with altitude.
Derive the formula that you may use. *(Delhi University, 1992)* [**Ans.** (*i*) 37.45 kN/m^2 (*ii*) 31.5 kN/m^2]

24. What are the gauge pressure and absolute pressure at a point 4 m below the free surface of a liquid of specific gravity 1.53, if atmospheric pressure is equivalent to 750 mm of mercury.
(Delhi University, 1992) [**Ans.** 60037 N/m^2 and 160099 N/m^2]

25. Find the gauge pressure and absolute pressure in N/m^2 at a point 4 m below the free surface of a liquid of sp. gr. 1.2, if the atmospheric pressure is equivalent to 750 mm of mercury.
(Delhi University, May 1998) [**Ans.** 47088 N/m^2 ; 147150 N/m^2]

26. A tank contains a liquid of specific gravity 0.8. Find the absolute pressure and gauge pressure at a point, which is 2 m below the free surface of the liquid. The atmospheric pressure head is equivalent to 760 mm of mercury. *(Delhi University, June 1996)* [**Ans.** 117092 N/m^2 ; 15696 N/m^2]

3

CHAPTER

Hydrostatic Forces on Surfaces

▶ 3.1 INTRODUCTION

This chapter deals with the fluids (*i.e.,* liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

▶ 3.2 TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▶ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F)**. The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

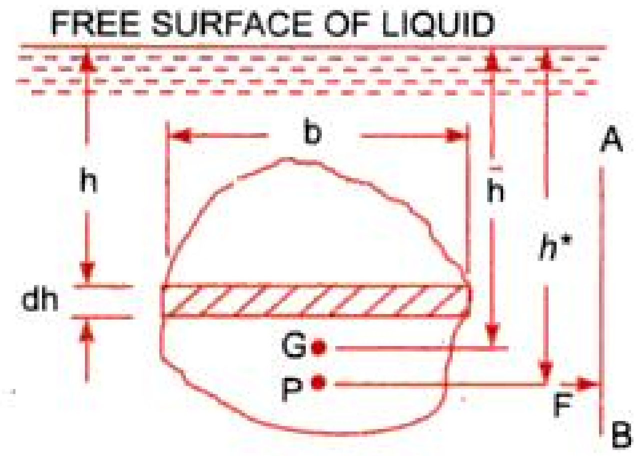


Fig. 3.1

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= \text{A} \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h} \qquad \text{...(3.1)}$$

For water the value of $\rho = 1000 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\{\because \ dF = \rho g h \times b \times dh\}$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \ bdh = dA)$$

But $\int h^2 dA = \int b h^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \qquad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_G in equation (3.4), we get

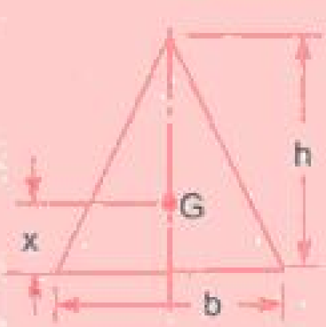
$$h^* = \frac{I_G + A\overline{h^2}}{Ah} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \text{...(3.5)}$$

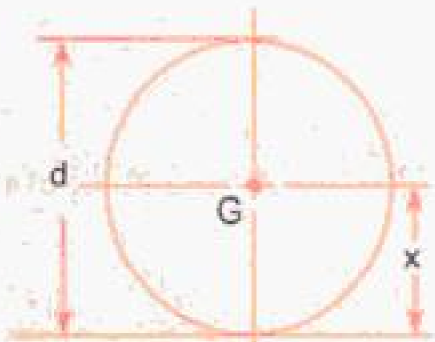
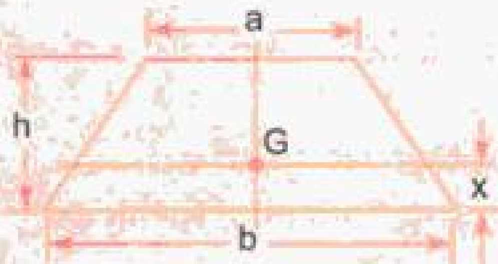
In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

$$A = 3 \times 2 = 6 \text{ m}^2, \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

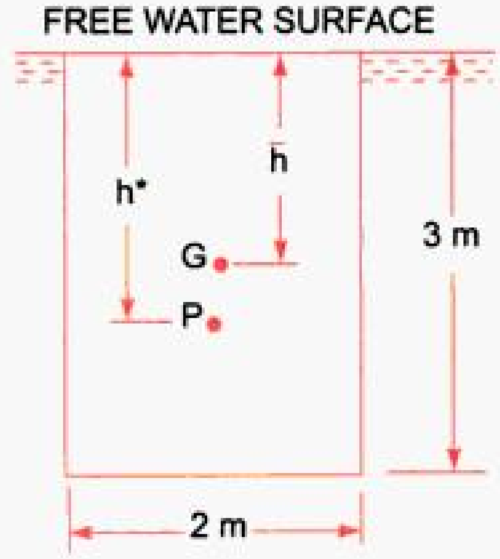


Fig. 3.2

$\therefore$
$$F = 1000 \times 9.81 \times 6 \times 1.5$$
$$= \mathbf{88290 \text{ N. Ans.}}$$

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.\ Ans.}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (*F*) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \qquad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440\ N.\ Ans.}$$

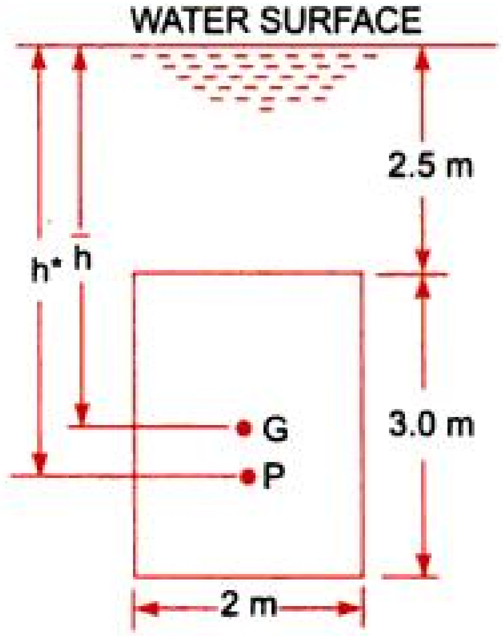


Fig. 3.3

Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \qquad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.\ Ans.}$$

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Solution. Given : Dia. of plate, $d = 1.5$ m

$$\therefore \text{ Area,} \qquad A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$

$$= \mathbf{52002.81\ N.\ Ans.}$$

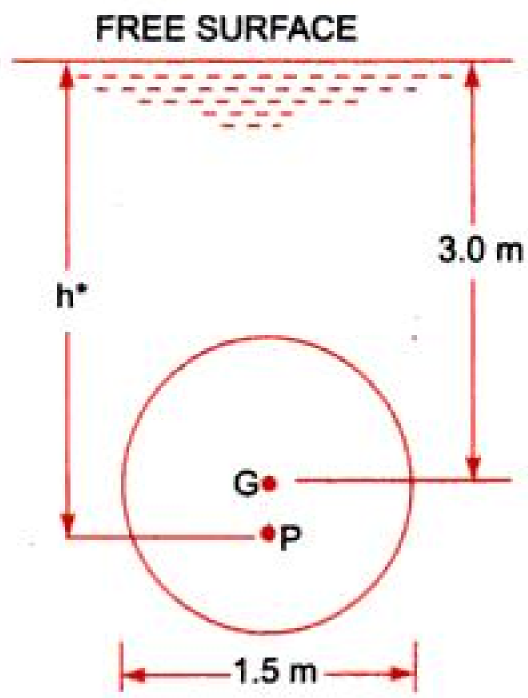


Fig. 3.4

Position of centre of pressure (h^*) is given by equation (3.5)

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$$\therefore \qquad h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$

$$= \mathbf{3.0468\ m.\ Ans.}$$

Problem 3.3 *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p + \dfrac{d^2}{12p}\right)$.

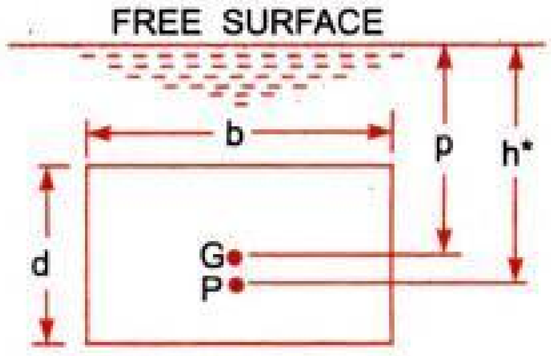


Fig. 3.5

Solution. Given :

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d \text{ m}^2$

Depth of C.G. from free surface

$$\bar{h} = p \text{ m}.$$

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + h, \text{ where } I_G = \frac{bd^2}{12}$$

$$\therefore \quad h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \frac{d^2}{12p} + p \quad \text{or} \quad \mathbf{p + \frac{d^2}{12}}. \textbf{ Ans.}$$

Problem 3.4 *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

(i) the force on the disc, and

(ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m. (A.M.I.E., Winter, 1977)

Solution. Given :

Dia. of opening, $d = 3$ m

$\therefore$ Area, $A = \dfrac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2.$

Depth of C.G., $\bar{h} = 4$ m

(*i*) Force on the disc is given by equation (3.1) as

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$$

$$= \mathbf{277368 \text{ N} = 277.368 \text{ kN}}. \textbf{ Ans.}$$

(*ii*) To find the torque required to maintain the disc in equilibrium, first calculate the point of application of force acting on the disc, *i.e.*, centre of pressure of the force F. The depth of centre of pressure (h^*) is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + h = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0 \qquad \left\{\because I_G = \frac{\pi}{64} d^4\right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 \text{ m}$$

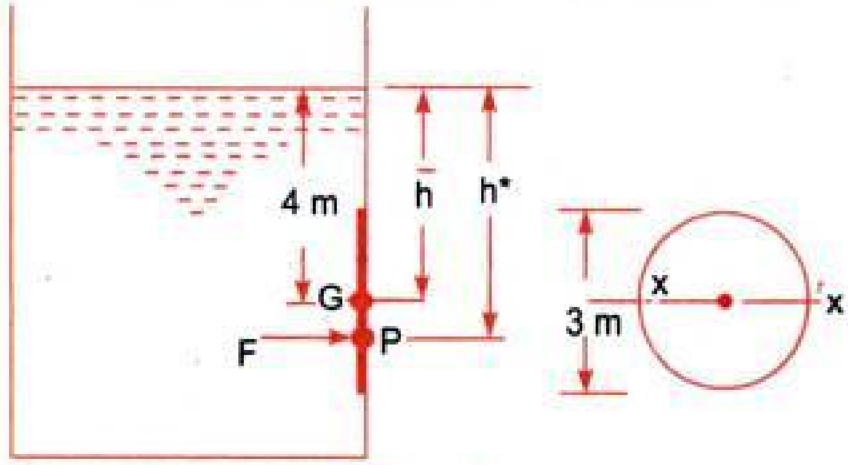


Fig. 3.6

The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

$$= F \times \left(h^* - \bar{h}\right) = 277368\ (4.14 - 4.0) = \mathbf{38831\ Nm.\ Ans.}$$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.* (Converted to SI Units, A.M.I.E., Winter, 1975)

Solution. Given :

Dia. of pipe, $d = 4$ m

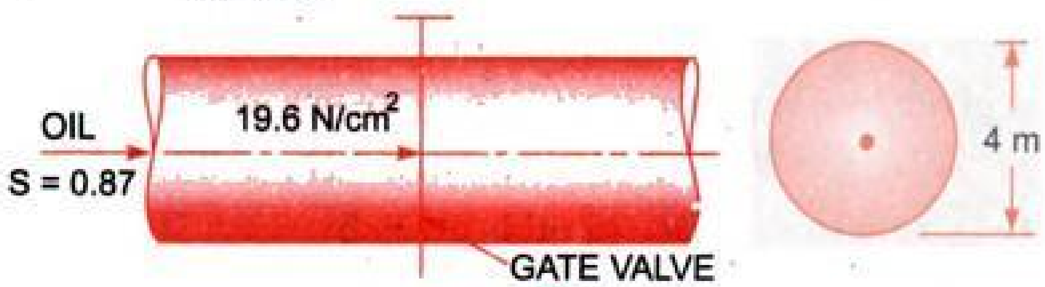


Fig. 3.7

$\therefore$ Area, $A = \dfrac{\pi}{4} \times 4^2 = 4\pi\ \text{m}^2$

Sp. gr. of oil, $S = 0.87$

$\therefore$ Density of oil, $\rho_0 = 0.87 \times 1000 = 870\ \text{kg/m}^3$

$\therefore$ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81\ \text{N/m}^3$

Pressure at the centre of pipe, $p = 19.6\ \text{N/cm}^2 = 19.6 \times 10^4\ \text{N/m}^2$

$$\therefore \quad \text{Pressure head at the centre} = \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988\ \text{m}$$

$\therefore$ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500\ N = 2.465\ MN.\ Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\,\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m.\ Ans.}$$

Or centre of pressure is below the centre of the pipe by a distance of 0.043 m. **Ans.**

Problem 3.6 *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

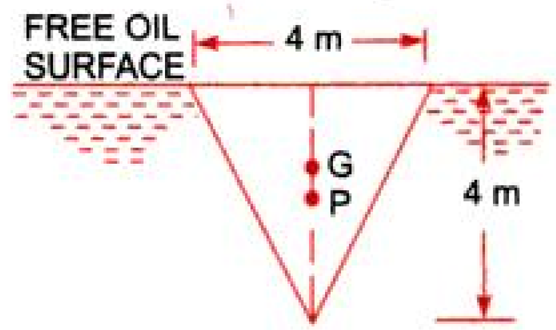


Fig. 3.8

Solution. Given :

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

$\therefore$ Area, $A = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8.0\ m^2$

Sp. gr. of oil, $S = 0.9$

$\therefore$ Density of oil, $\rho = 900\ kg/m^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33\ m.$$

Total pressure (F) is given by $F = \rho g A \bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33\ N = \mathbf{9597.6.\ N.\ Ans.}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11\ m^4$$

$$\therefore \quad h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99\ m.\ Ans.}$$

Problem 3.7 *A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.* (A.M.I.E., May, 1975)

Solution. Given :

Width of gate, $b = 2$ m

Depth of gate, $d = 1.2$ m

$\therefore$ Area, $A = b \times d = 2 \times 1.2 = 2.4\ m^2$

Sp. gr. of liquid $= 1.45$

$\therefore$ Density of liquid, $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^3

Let F_1 = Force exerted by the fluid of sp. gr. 1.45 on gate

F_2 = Force exerted by water on the gate.

The force F_1 is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$

where $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^2

$\bar{h}_1$ = Depth of C.G. of gate from free surface of liquid

$$= 1.5 + \frac{1.2}{2} = 2.1 \text{ m.}$$

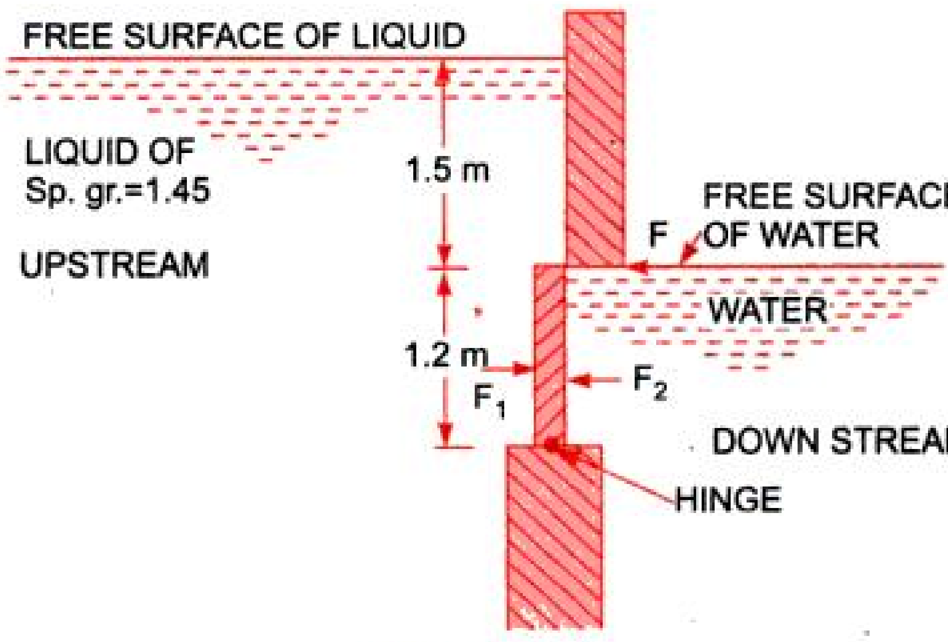


Fig. 3.9

$$\therefore \qquad F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691 \text{ N}$$

Similarly, $F_2 = \rho_2 g . A \bar{h}_2$

where $\rho_2 = 1{,}000$ kg/m^3

$\bar{h}_2$ = Depth of C.G. of gate from free surface of water

$$= \frac{1}{2} \times 1.2 = 0.6 \text{ m}$$

$$\therefore \qquad F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126 \text{ N}$$

(*i*) **Resultant force on the gate** = $F_1 - F_2$ = 71691 − 14126 = **57565 N. Ans.**

(*ii*) **Position of centre of pressure of resultant force.** The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h_1^* = \frac{I_G}{A\bar{h}_1} + \bar{h}_1$$

where $I_G = \dfrac{bd^3}{12} = \dfrac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$

$$\therefore \qquad h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

$\therefore$ Distance of F_1 from hinge

$$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429 \text{ m}$$

The force F_2 will be acting at a depth of h_2^* from free surface of water and is given by

$$h_2^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

where $I_G = 0.288 \text{ m}^4$, $\bar{h}_2 = 0.6$ m, $A = 2.4 \text{ m}^2$,

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of F_2 from hinge $= 1.2 - 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

= 0.578 m above the hinge. Ans.

(*iii*) **Force at the top of gate which is capable of opening the gate.** Let F is the force required on the top of the gate to open it as shown in Fig. 3.9. Taking the moments of F, F_1 and F_2 about the hinge, we get

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$$

or
$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

= 27725.5 N. Ans.

Problem 3.8 *A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 m wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.*

Solution. Given :

Width at top $= 16$ m

Width at bottom $= 10$ m

Depth, $d = 6$ m

Area of trapezoidal $ABCD$,

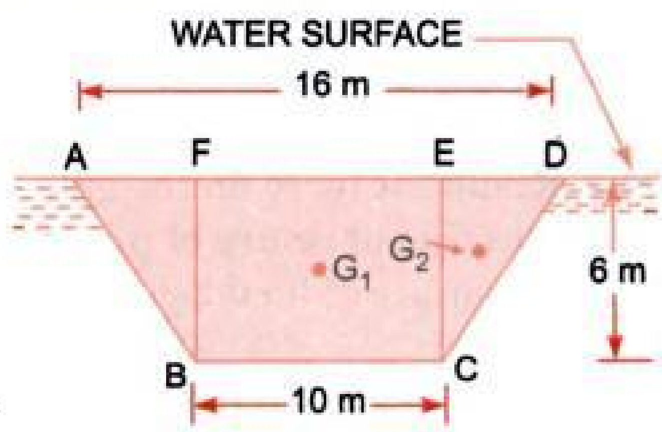


Fig. 3.10

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10 + 16)}{2} \times 6 = 78 \text{ m}^2$$

Depth of C.G. of trapezoidal area $ABCD$ from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16-10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \text{ m from water surface.}$$

(*i*) **Total Pressure (F).** Total pressure, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 78 \times 2.769 \text{ N}$$
$$= 2118783 \text{ N} = \mathbf{2.118783 \text{ MN. Ans.}}$$

(*ii*) **Centre of Pressure (h*).** Centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of trapezoidal *ABCD* about its C.G.

Let I_{G_1} = M.O.I. of rectangle *FBCE* about its C.G.

I_{G_2} = M.O.I. of two Δs *ABF* and *ECD* about its C.G.

Then
$$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ m}^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1.

∴ M.O.I. of the rectangle about the axis passing through the C.G. of the trapezoidal I_{G_1} + Area of rectangle $\times {x_1}^2$

where x_1 is distance between the C.G. of rectangle and C.G. of trapezoidal

$$= (3.0 - 2.769) = 0.231 \text{ m}$$

∴ M.O.I. of *FBCE* passing through C.G. of trapezoidal

$$= 180 + 10 \times 6 \times (0.231)^2 = 180 + 3.20 = 183.20 \text{ m}^4$$

Now
$$I_{G_2} = \text{M.O.I. of } \Delta ABD \text{ in Fig. 3.11 about } G_2 = \frac{bd^3}{36}$$

$$= \frac{(16-10) \times 6^3}{36} = 36 \text{ m}^4$$

The distance between the C.G. of triangle and C.G. of trapezoidal

$$= (2.769 - 2.0) = 0.769$$

∴ M.O.I. of the two Δs about an axis passing through C.G. of trapezoidal

$$= I_{G_2} + \text{Area of triangles} \times (.769)^2$$

$$= 36.0 + \frac{6 \times 6}{2} \times (.769)^2$$

$$= 36.0 + 10.64 = 46.64$$

∴ I_G = M.O.I. of trapezoidal about its C.G.

= M.O.I. of rectangle about the C.G. of trapezoidal

+ M.O.I. of triangles about the C.G. of the trapezoidal

$= 183.20 + 46.64 = 229.84 \text{ m}^4$

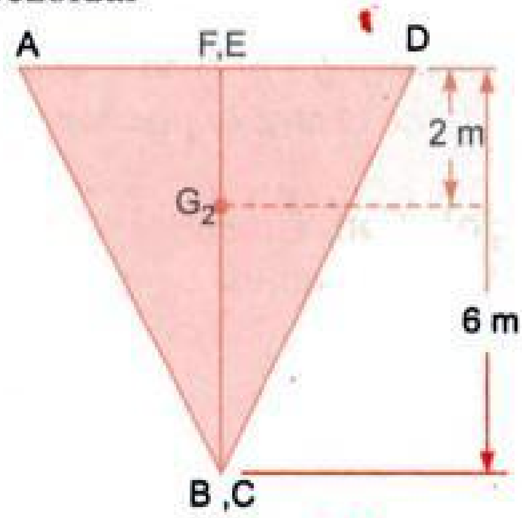


Fig. 3.11

∴
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $A = 78$, $\bar{h} = 2.769$

$$h^* = \frac{229.84}{78 \times 2.769} + 2.769 = 1.064 + 2.769 = \mathbf{3.833 \text{ m. Ans.}}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface *AD* is given by (refer to Table 3.1 on page 69)

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$$

$$= \frac{(2\times 10+16)}{(10+16)} \times \frac{6}{3} \qquad (\because \quad a = 10, b = 16 \text{ and } h = 6)$$

$$= \frac{36}{26} \times 2 = 2.769 \text{ m}$$

This is also equal to the distance of the C.G. of the trapezoidal from free surface of water.

$$\bar{h} = 2.769 \text{ m}$$

$\therefore$ Total pressure, $\quad F = \rho g A \bar{h} \qquad (\because \quad A = 78)$

$$= 1000 \times 9.81 \times 78 \times 2.769 \text{ N} = \mathbf{2118783 \text{ N. Ans.}}$$

Centre of Pressure

$$(h^*) = \frac{I_G}{A\bar{h}} + \bar{h}$$

Now I_G from Table 3.1 is given by,

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(10^2 + 4\times 10 \times 16 + 16^2)}{36(10+16)} \times 6^3$$

$$= \frac{(100+640+256)}{36 \times 26} \times 216 = 229.846 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{229.846}{78 \times 2.769} + 2.769 \qquad (\because \quad A = 78 \text{ m}^2)$$

$$= \mathbf{3.833 \text{ m. Ans.}}$$

Problem 3.9 *A trapezoidal channel 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine :*

(i) the total pressure, and

(ii) the centre of pressure on the vertical gate closing the channel when it is full of water.

(A.M.I.E., Summer, 1978)

Solution. Given :

Width at bottom $= 2$ m

Depth, $d = 1$ m

Side slopes $= 1 : 1$

$\therefore$ Top width, $AD = 2 + 1 + 1 = 4$ m

Area of rectangle $FBEC$, $A_1 = 2 \times 1 = 2 \text{ m}^2$

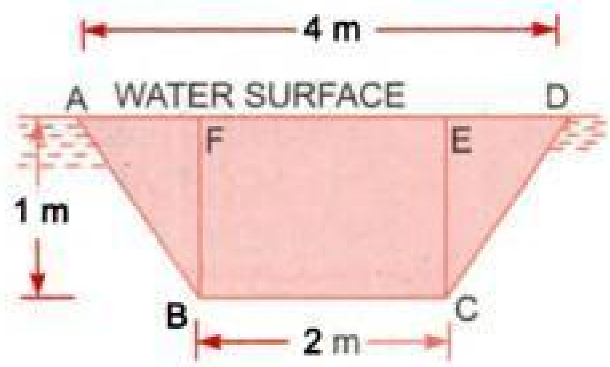


Fig. 3.12

Area of two triangles ABF and ECD, $A_2 = \frac{(4-2)}{2} \times 1 = 1 \text{ m}^2$

$\therefore$ Area of trapezoidal $ABCD$, $A = A_1 + A_2 = 2 + 1 = 3 \text{ m}^2$

Depth of C.G. of rectangle $FBEC$ from water surface,

$$\bar{h}_1 = \frac{1}{2} = 0.5 \text{ m}$$

Depth of C.G. of two triangles *ABF* and *ECD* from water surface,

$$\bar{h}_2 = \frac{1}{3} \times 1 = \frac{1}{3} \text{ m}$$

$\therefore$ Depth of C.G. of trapezoidal *ABCD* from free surface of water

$$\bar{h} = \frac{A_1 \times \bar{h}_1 + A_2 \times \bar{h}_2}{(A_1 + A_2)} = \frac{2 \times 0.5 + 1 \times 0.33333}{(2+1)} = .44444$$

(*i*) **Total Pressure (F).** Total pressure F is given by

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 3.0 \times 0.44444 = \mathbf{13079.9\ N.\ Ans.}$$

(*ii*) **Centre of Pressure (h*).** M.O.I. of rectangle *FBCE* about its C.G.,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = \frac{1}{6} \text{ m}^4$$

M.O.I. of *FBCE* about an axis passing through the C.G. of trapezoidal

or $$I_{G_1}{}^* = I_{G_1} + A_1 \times [\text{Distance between C.G. of rectangle and C.G. of trapezoidal}]^2$$

$$= \frac{1}{6} + 2 \times \left[\bar{h}_1 - \bar{h}\right]^2$$

$$= \frac{1}{6} + 2 \times [0.5 - .4444]^2 = .1666 + .006182 = 0.1727$$

M.O.I. of the two triangles *ABF* and *ECD* about their C.G.,

$$I_{G_2} = \frac{bd^3}{36} = \frac{(1+1) \times 1^3}{36} = \frac{2}{36} = \frac{1}{18} \text{ m}^4.$$

M.O.I. of the two triangles about the C.G. of trapezoidal,

$$I_{G_2}{}^* = I_{G_1} + A_2 \times [\text{Distance between C.G. of triangles and C.G. of trapezoidal}]^2$$

$$= \frac{1}{18} + 1 \times \left[\bar{h} - \bar{h}_2\right]^2 = \frac{1}{18} + 1 \times \left[.4444 - \frac{1}{3}\right]^2$$

$$= \frac{1}{18} + (.1111)^2 = 0.0555 + (.1111)^2$$

$$= .0555 + 0.01234 = 0.06789 \text{ m}^4$$

$\therefore$ M.O.I. of the trapezoidal about its C.G.

$$I_G = I_{G_1}{}^* + I_{G_2}{}^* = .1727 + .06789 = 0.24059 \text{ m}^4$$

Then centre of pressure (h^*) on the vertical trapezoidal,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.24059}{3 \times .4444} + .4444 = 0.18046 + .4444 = 0.6248$$

$$\simeq \mathbf{0.625\ m.\ Ans.}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface AD is given by (refer to Table 3.1 on page 69).

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3} = \frac{(2\times 2+4)}{(2+4)} \times \frac{1}{3} \qquad (\because \quad a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$$\therefore \qquad \bar{h} = x = 0.444 \text{ m}$$

$\therefore$ Total pressure,

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3.0 \times .444 \qquad (\because \quad A = 3.0)$$

$$= \mathbf{13079 \text{ N. Ans.}}$$

Centre of pressure,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G from Table 3.1 is given by

$$I_G = \frac{(a^2+4ab+b^2)}{36(a+b)} \times h^3 = \frac{(2^2+4\times2\times4+4^2)}{36(2+4)} \times 1^3 = \frac{52}{36\times6} = 0.2407 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{0.2407}{3.0\times.444} + .444 = \mathbf{0.625 \text{ m. Ans.}}$$

Problem 3.10 *A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure.* (A.M.I.E., Summer, 1986)

Solution. Given : Diagonals of aperture, $AC = BD = 2$ m

$\therefore$ Area of square aperture, A = Area of ΔACB + Area of ΔACD

$$= \frac{AC\times BO}{2} + \frac{AC\times OD}{2} = \frac{2\times1}{2} + \frac{2\times1}{2} = 1 + 1 = 2.0 \text{ m}^2$$

Sp. gr. of liquid $= 1.15$

$\therefore$ Density of liquid, $\rho = 1.15 \times 1000 = 1150 \text{ kg/m}^3$

Depth of centre of aperture from free surface,

$$\bar{h} = 1.5 \text{ m.}$$

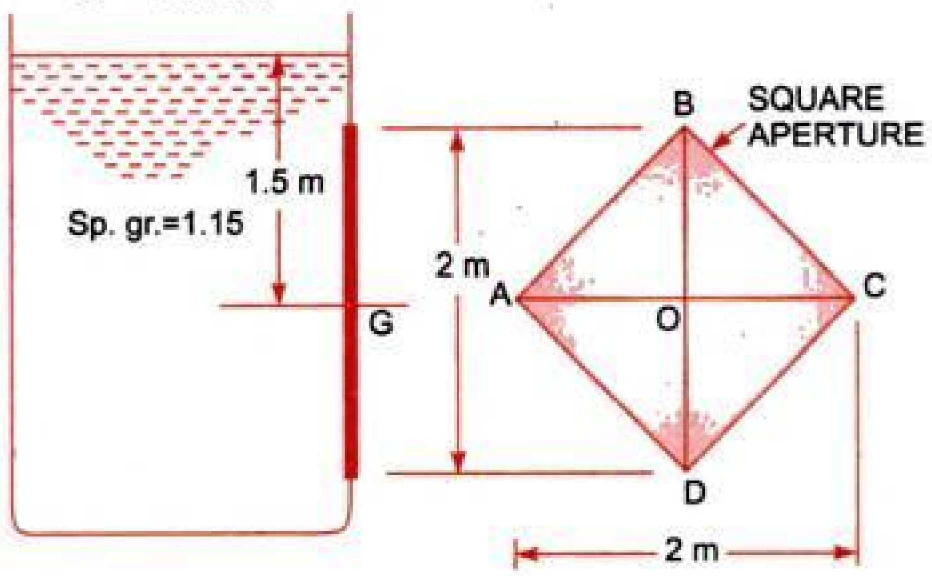


Fig. 3.13

(*i*) The thrust on the plate is given by

$$F = \rho g A\bar{h} = 1150 \times 9.81 \times 2 \times 1.5 = \mathbf{33844.5.\ Ans.}$$

(*ii*) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of *ABCD* about diagonal *AC*

= M.O.I. of triangle *ABO* about *AC* + M.O.I. of triangle *ACD* about *AC*

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12} \qquad \left(\because \text{ M.O.I. of a triangle about its base} = \frac{bh^3}{12}\right)$$

$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}\ \text{m}^4$$

$$\therefore \qquad h^* = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = \frac{1}{3 \times 2 \times 1.5} + 1.5 = \mathbf{1.611\ m.\ Ans.}$$

Problem 3.11 *A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1 m height. Calculate :*

(i) total pressure on one side of the tank,

(ii) the position of centre of pressure for one side of the tank, which is 2 m wide.

Solution. Given :

Depth of water = 0.5 m

Depth of liquid = 1 m

Sp. gr. of liquid = 0.8

Density of liquid, $\rho_1 = 0.8 \times 1000 = 800$ kg/m^3

Density of water, $\rho_2 = 1000$ kg/m^3

Width of tank = 2 m

(*i*) **Total pressure on one side** is calculated by drawing pressure diagram, which is shown in Fig. 3.14.

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on *D* (or *DE*), $p_D = \rho_1 g.h_1$

$= 800 \times 9.81 \times 1.0 = 7848$ N/m^2

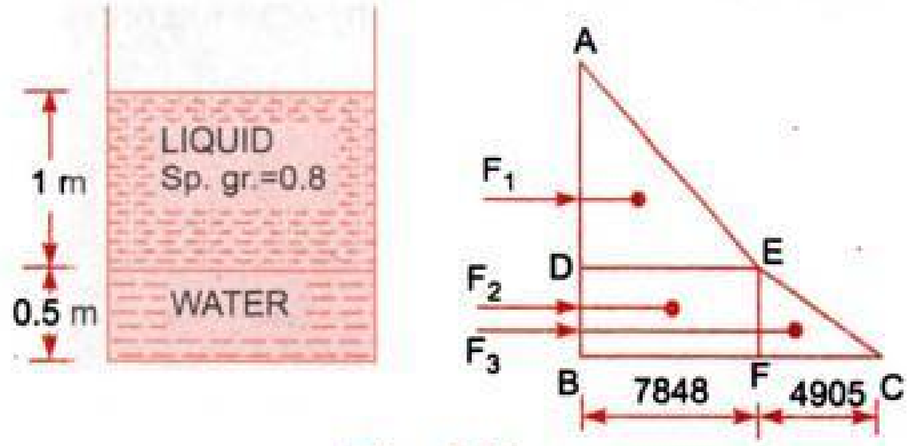


Fig. 3.14

Intensity of pressure on base (or *BC*), $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753$ N/m^2

Now force F_1 = Area of ΔADE × Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848\ \text{N}$$

Force $\qquad F_2$ = Area of rectangle $DBFE$ × Width of tank

$= 0.5 \times 7848 \times 2 = 7848$ N

F_3 = Area of ΔEFC × Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

∴ Total pressure, $\qquad F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 =$ **18148.5 N. Ans.**

(*ii*) **Centre of Pressure (h*).** Taking the moments of all force about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 (AD + \frac{1}{2} BD) + F_3[AD + \frac{2}{3} BD]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5\right)$$

$$= 5232 + 9810 + 3270 = 18312$$

$$\therefore \qquad h^* = \frac{18312}{18148.5} = \textbf{1.009 m from top. Ans.}$$

Problem 3.12 *A cubical tank has sides of 1.5 m. It contains water for the lower 0.6 m depth. The upper remaining part is filled with oil of specific gravity 0.9. Calculate for one vertical side of the tank :*

(a) total pressure, and

(b) position of centre of pressure. (A.M.I.E., Winter, 1987)

Solution. Given :

Cubical tank of sides 1.5 m means the dimensions of the tank are 1.5 m × 1.5 m × 1.5 m.

Depth of water $= 0.6$ m

Depth of liquid $= 1.5 - 0.6 = 0.9$ m

Sp. gr. of liquid $= 0.9$

Density of liquid, $\rho_1 = 0.9 \times 1000 = 900$ kg/m^3

Density of water, $\rho_2 = 1000$ kg/m^3

(*a*) **Total pressure** on one vertical side is calculated by drawing pressure diagram, which is shown in Fig. 3.15.

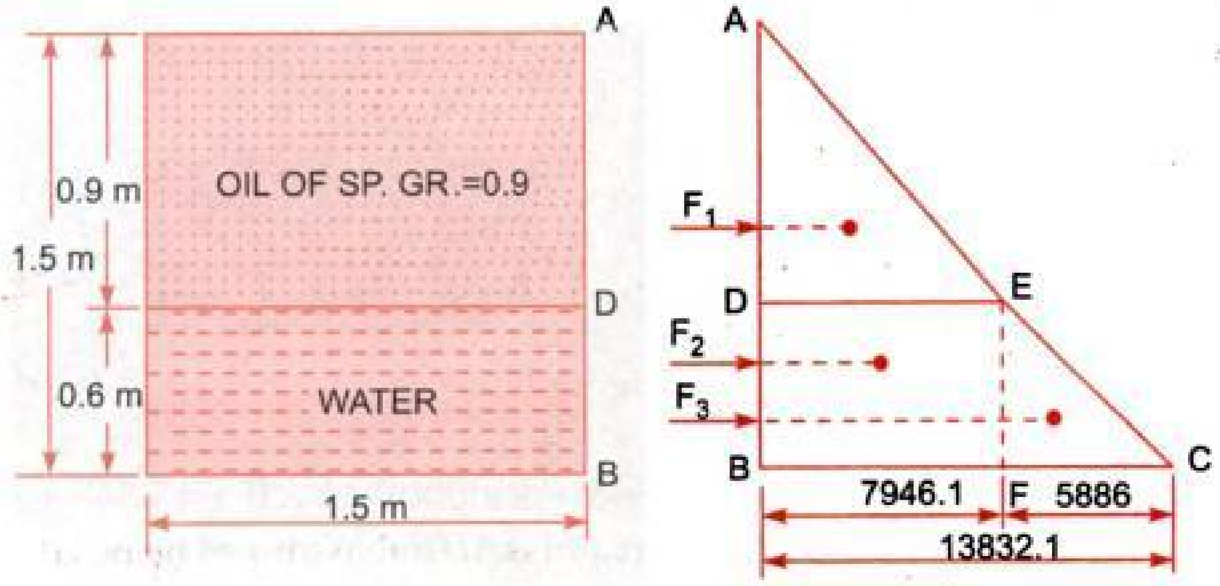


Fig. 3.15

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Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho g h$

$\therefore$ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho g h \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho g h dA$

But from Fig. 3.18,
$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$$

$\therefore$
$$h = y \sin\theta$$

$\therefore$
$$F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$$

But
$$\int y dA = A\bar{y}$$

where $\bar{y}$ = Distance of C.G. from axis O-O

$\therefore$
$$F = \rho g \sin\theta\, \bar{y} \times A$$
$$= \rho g A \bar{h} \qquad (\because \bar{h} = \bar{y}\sin\theta) \quad ...(3.6)$$

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho g h dA$

$$= \rho g y \sin\theta\, dA \qquad [h = y\sin\theta]$$

Moment of the force, dF, about axis O-O

$$= dF \times y = \rho g y \sin\theta\, dA \times y = \rho g \sin\theta\, y^2 dA$$

Sum of moments of all such forces about O-O

$$= \int \rho g \sin\theta\, y^2\, dA = \rho g \sin\theta \int y^2\, dA$$

But
$$\int y^2\, dA = \text{M.O.I. of the surface about } O\text{-}O = I_0$$

$\therefore$ Sum of moments of all forces about O-O $= \rho g \sin\theta\, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$$= F \times y^* \quad ...(3.8)$$

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$$F \times y^* = \rho g \sin\theta\, I_0$$

or
$$y^* = \frac{\rho g \sin\theta\, I_0}{F} \quad ...(3.9)$$

Now
$$y^* = \frac{h^*}{\sin\theta},\ F = \rho g A \bar{h}$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

(*ii*) **Position of centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64} [d^4 - d_0^4] = \frac{\pi}{64} [3^4 - 1.5^4] \text{ m}^4$

$$A = \frac{\pi}{4} [d^2 - d_0^2] = \frac{\pi}{4} [3^2 - 1.5^2] \text{ m}^2$$

$$\sin \theta = \frac{2.5}{3} \text{ and } \bar{h} = 2.75$$

$$\therefore \qquad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right] \times \left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right] \times 2.75} + 2.75$$

$$= \frac{\frac{1}{16}\left[3^2 + 1.5^2\right] \times \left(\frac{2.5}{3}\right)^2}{2.75} + 2.75 = \frac{1 \times 11.25 \times 6.25}{16 \times 2.75 \times 9} + 2.75$$

$= 0.177 + 2.75 =$ **2.927 m. Ans.**

Problem 3.16 *A circular plate 3 metre diameter is submerged in water as shown in Fig. 3.21. Its greatest and least depths are below the surfaces being 2 metre and 1 metre respectively. Find : (i) the total pressure on front face of the plate, and (ii) the position of centre of pressure.*

(A.M.I.E., Winter, 1983)

Solution. Given :

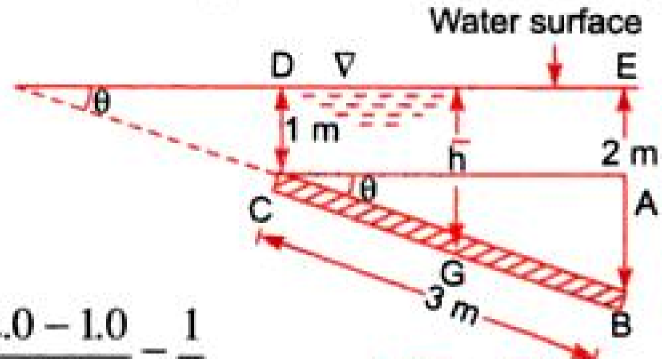

Fig. 3.21

Dia. of plate, $d = 3.0$ m

$\therefore$ Area, $A = \frac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance, $DC = 1$ m, $BE = 2$ m

In ΔABC, $\sin \theta = \frac{AB}{AC} = \frac{BE - AE}{BC} = \frac{BE - DC}{BC} = \frac{2.0 - 1.0}{3.0} = \frac{1}{3}$

The centre of gravity of the plate is at the middle of BC, *i.e.*, at a distance 1.5 m from C.

The distance of centre of gravity from the free surface of the water is given by

$$\bar{h} = CD + CG \sin \theta = 1.0 + 1.5 \times \tfrac{1}{3} \qquad (\because \sin \theta = \tfrac{1}{3})$$

$= 1.5$ m.

(*i*) Total pressure on the front face of the plate is given by

$$F = \rho g A \bar{h}$$

$= 1000 \times 9.81 \times 7.0685 \times 1.5 =$ **104013 N. Ans.**

Let the distance of the centre of pressure from the free surface of the water be h^*. Then using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

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$$\therefore \qquad h^* = \frac{2h^3}{9 \times \sqrt{3}} \times \frac{\sin^2 60^\circ}{\frac{2h}{\sqrt{3}} \times \frac{h}{2}} + \frac{h}{2} = \frac{2h^3 \times \frac{3}{4}}{9h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{2h}{3}$$

Equating the two values of h^*,

$$h - 3 = \frac{2h}{3} \quad \text{or} \quad h - \frac{2h}{3} = 3 \quad \text{or} \quad \frac{h}{3} = 3$$

$$\therefore \qquad h = 3 \times 3 = 9 \text{ m}$$

$\therefore$ Height of water for tipping the gate = **9 m. Ans.**

Problem 3.20 *A rectangular sluice gate AB, 2 m wide and 3 m long is hinged at A as shown in Fig. 3.25. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to the gate is 343350 N. Find the height of the water 'h' which will just cause the gate to open. The centre of gravity of the weight and gate is at G.*

Solution. Given :

Width of gate, $b = 2$ m ; Length of gate $L = 3$ m

$\therefore$ Area, $A = 2 \times 3 = 6 \text{ m}^2$

Weight of gate and $W = 343350$ N

Angle of inclination, $\theta = 45^\circ$

Let h is the required height of water.

Depth of C.G. of the gate and weight = $\bar{h}$

From Fig. 3.25 (a),

$$\bar{h} = h - ED = h - (AD - AE)$$

$$= h - (AB \sin\theta - EG \tan\theta) \qquad \left\{\because \tan\theta = \frac{AE}{EG} \therefore AE = EG \tan\theta\right\}$$

$$= h - (3 \sin 45^\circ - 0.6 \tan 45^\circ)$$

$$= h - (2.121 - 0.6) = (h - 1.521) \text{ m}$$

The total pressure force, F is given by

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 6 \times (h - 1.521)$$

$$= 58860\ (h - 1.521) \text{ N.}$$

The total force F is acting at the centre of pressure as shown in Fig. 3.25 (b) at H. The depth of H from free surface is given by h^* which is equal to

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = \frac{54}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5 \times \sin^2 45}{6 \times (h - 1.521)} + (h - 1.521) = \frac{0.375}{(h - 1.521)} + (h - 1.521) \text{ m}$$

= weight of liquid supported by the curved surface upto free surface of liquid. ...(3.18)

In Fig. 3.28, the curved surface AB is not supporting any fluid. In such cases, F_y is equal to the weight of the imaginary liquid supported by AB upto free surface of liquid. The direction of F_y will be taken in upward direction.

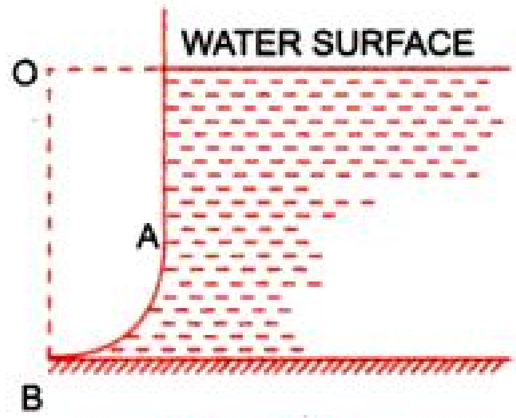

Fig. 3.28

Problem 3.22 *Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.29. Take the width of the gate as unity.*

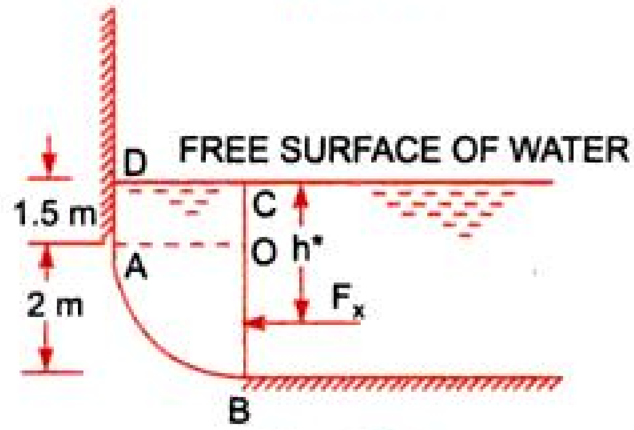

Fig. 3.29

Solution. Given :

Width of gate = 1.0 m

Radius of the gate = 2.0 m

∴ Distance $AO = OB = 2$ m

Horizontal force, F_x exerted by water on gate is given by equation (3.17) as

F_x = Total pressure force on the projected area of curved surface AB on vertical plane

= Total pressure force on OB

{projected area of curved surface on vertical plane = $OB \times 1$}

$= \rho g A\bar{h}$

$= 1000 \times 9.81 \times 2 \times 1 \times \left(1.5 + \frac{2}{2}\right)$ {∵ Area of $OB = A = BO \times 1 = 2 \times 1 = 2$,

$\bar{h}$ = Depth of C.G. of OB from free surface = $1.5 + \frac{2}{2}$ }

$F_x = 9.81 \times 2000 \times 2.5 =$ **49050 N. Ans.**

The point of application of F_x is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where I_G = M.O.I. of OB about its C.G. $= \dfrac{bd^3}{12} = \dfrac{1 \times 2^3}{12} = \dfrac{2}{3}$ m^4

$$\therefore \qquad h^* = \frac{\frac{2}{3}}{2 \times 2.5} + 2.5 = \frac{1}{7.5} + 2.5 \text{ m}$$

= 0.1333 + 2.5 = 2.633 m from free surface.

Vertical force, F_y, exerted by water is given by equation (3.18)

F_y = Weight of water supported by AB upto free surface

= Weight of portion $DABOC$

= Weight of $DAOC$ + Weight of water AOB

= ρg [Volume of $DAOC$ + Volume of AOB]

$$= 1000 \times 9.81 \left[AD \times AO \times 1 + \frac{\pi}{4}(AO)^2 \times 1\right]$$

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$$F_{y_1} = \text{weight of water enclosed by } ABCOA$$

$$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = \mathbf{123276\ N.}$$

Right Side of the Cylinder

$$F_{x_2} = \rho g A_2 \bar{h}_2 = \text{Force on vertical area } CO$$

$$= 1000 \times 9.81 \times 2 \times 2 \times \frac{2}{2} \left\{\because\ A_2 = CO \times 1 = 2 \times 1 = 2\ \text{m}^2, \bar{h}_2 = \frac{2}{2} = 1.0\right\}$$

$$= 39240\ \text{N}$$

$$F_{y_2} = \text{Weight of water enclosed by } DOCD$$

$$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638\ \text{N}$$

$\therefore$ Resultant force in the direction of x,

$$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720\ \text{N}$$

Resultant force in the direction of y,

$$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914\ \text{N}$$

(*i*) **Resultant force,** F is given as

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = \mathbf{219206\ N.\ Ans.}$$

(*ii*) **Direction of resultant force** is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$$

$$\therefore \qquad \theta = \mathbf{57° \ 31'.\ Ans.}$$

(*iii*) **Location of the resultant force**

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67$ m from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33$ m from free surface on the right side of the cylinder. The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$$F_x \times y = F_{x_1}\,[4 - 2.67] - F_{x_2}\,[2 - 1.33]$$

or
$$117720 \times y = 156960 \times 1.33 - 39240 \times .67 = 208756.8 - 26290.8 = 182466$$

$$\therefore \qquad y = \frac{182466}{117720} = 1.55 \text{ m from the bottom.}$$

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488$ m from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488$ m from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

(*iii*) **Resultant thrust exerted by water**

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{397305 + 353160} = \mathbf{531574\ N.\ Ans.}$$

Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = \mathbf{41.63°.\ Ans.}$$

Problem 3.31 *A cylinder 3 m in diameter and 4 m long retains water on one side. The cylinder is supported as shown in Fig. 3.39. Determine the horizontal reaction at A and the vertical reaction at B. The cylinder weighs 196.2 kN. Ignore friction.*

Solution. Given :

Dia. of cylinder $= 3$ m

Length of cylinder $= 4$ m

Weight of cylinder, $W = 196.2$ kN $= 196200$ N

Horizontal force exerted by water

F_x = Force on vertical area *BOC*

$= \rho g A\bar{h}$

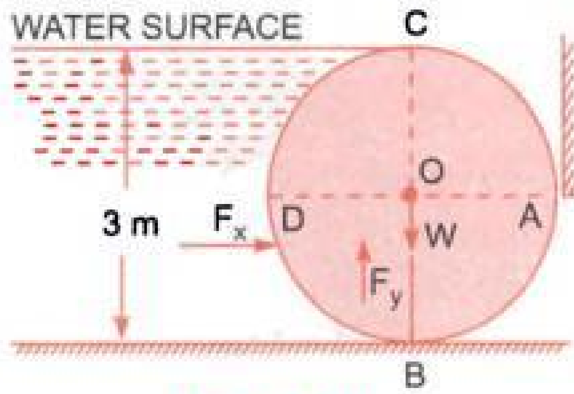

Fig. 3.39

where $A = BOC \times l = 3 \times 4 = 12\ \text{m}^2,\ \bar{h} = \frac{1}{2} \times 3 = 1.5$ m

$F_x = 1000 \times 9.81 \times 12 \times 1.5 = 176580$ N

The vertical force exerted by water

F_y = Weight of water enclosed in *BDCOB*

$$= \rho g \times \left(\frac{\pi}{2}R^2\right) \times l = 1000 \times 9.81 \times \frac{\pi}{2} \times (1.5)^2 \times 4 = 138684 \text{ N}$$

Force F_y is acting in the upward direction.

For the equilibrium of cylinder

Horizontal reaction at $A = F_x = 176580$ N

Vertical reaction at B = Weight of cylinder $- F_y$

$= 196200 - 138684 = \mathbf{57516\ N.\ Ans.}$

▶ 3.7 TOTAL PRESSURE AND CENTRE OF PRESSURE ON LOCK GATES

Lock gates are the devices used for changing the water level in a canal or a river for navigation. Fig. 3.40 shows plan and elevation of a pair of lock gates. Let *AB* and *BC* be the two lock gates. Each gate is supported on two hinges fixed on their top and bottom at the ends *A* and *C*. In the closed position, the gates meet at *B*.

Let F = Resultant force due to water on the gate *AB* or *BC* acting are right angles to the gate

R = Reaction at the lower and upper hinge

P = Reaction at the common contact surface of the two gates and acting perpendicular to the contact surface.

Let the force *P* and *F* meet at *O*. Then the reaction *R* must pass through *O* as the gate *AB* is in the equilibrium under the action of three forces. Let θ is the inclination of the lock gate with the normal to the side of the lock.

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Width of lock $= 10$ m

$\therefore$ Width of each lock $= \dfrac{5}{\cos 30}$ or $l = 5.773$ m

Depth of water on upstream side, $H_1 = 8$ m

Depth of water on downstream side, $H_2 = 4$ m

(*i*) **Water pressure on upstream side**

$$F_1 = \rho g A_1 \bar{h}_1$$

where $A_1 = l \times H_1 = 5.773 \times 8 = 46.184$ m, $\bar{h}_1 = \dfrac{H_1}{2} = \dfrac{8}{2} = 4.0$ m

$$F_1 = 1000 \times 9.81 \times 46.184 \times 4.0 = 1812260 \text{ N} = 1812.26 \text{ kN}$$

Water pressure on downstream side,

$$F_2 = \rho g A_2 \bar{h}_2$$

where $A_2 = l \times H_2 = 5.773 \times 4 = 23.092$ m, $\bar{h}_2 = \dfrac{4}{2} = 2.0$

$$F_2 = 1000 \times 9.81 \times 23.092 \times 2.0 = 453065 \text{ N} = 453.065 \text{ kN}$$

$\therefore$ Resultant water pressure

$$= F_1 - F_2 = 1812.26 - 453.065 = 1359.195 \text{ kN}$$

(*ii*) **Reaction between the gates AB and AC.** The reaction (P) between the gates AB and AC is given by equation (3.20) as

$$F = \frac{F}{2 \sin \theta} = \frac{1359.195}{2 \times \sin 30} = \mathbf{1359.195 \text{ kN. Ans.}}$$

(*iii*) **Force of each hinge.** If R_T and R_B are the reactions at the top and bottom hinges then

$$R_T + R_B = R$$

But from equation (3.19),$R = P = 1359.195$

$$\therefore \quad R_T + R_B = 1359.195$$

The force F_1 is acting at $\dfrac{H_1}{3} = \dfrac{8}{3} = 2.67$ m from bottom and F_2 at $\dfrac{H_2}{3} = \dfrac{4}{3} = 1.33$ m from bottom. The resultant force F will act at a distance x from bottom given by

$$F \times x = F_1 \times 2.67 - F_2 \times 1.33$$

or

$$x = \frac{F_1 \times 2.67 - F_2 \times 1.33}{F} = \frac{1812.26 \times 2.67 - 453.065 \times 1.33}{1359.195}$$

$$= \frac{4838.734 - 602.576}{1359.195} = 3.116 = 3.11 \text{ m}$$

Hence R is also acting at a distance 3.11m from bottom.

Taking moments of R_T and R about the bottom hinge

$$R_T \times [6.0 - 1.0] = R \times (x - 1.0)$$

$$\therefore \quad R_T = \frac{Rx\,(x - 1.0)}{5.0} = \frac{1359.195 \times 2.11}{5.0} = 573.58 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 1359.195 - 573.58$$

$$= \mathbf{785.615 \text{ kN. Ans.}}$$

Problem 3.34 *A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of 2.4 m/s². The length, width and depth of the tank are 6 m, 2.5 m and 2 m respectively. If the depth of water in the tank is 1 m and tank is open at the top then calculate :*

(*i*) *the angle of the water surface to the horizontal,*

(*ii*) *the maximum and minimum pressure intensities at the bottom,*

(*iii*) *the total force due to water acting on each end of the tank.*

Solution. Given :

Constant acceleration, $a = 2.4$ m/s².

Length = 6 m ; Width = 2.5 m and depth = 2 m.

Depth of water in tank, $h = 1$ m

(*i*) **The angle of the water surface to the horizontal**

Let θ = the angle of water surface to the horizontal

Using equation (3.20), we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

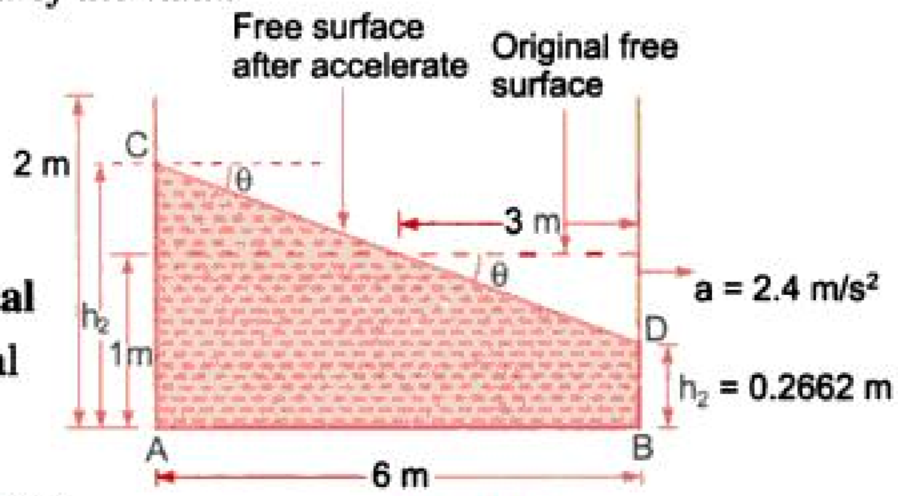

Fig. 3.45

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig. 3.45)

$\therefore$ $\tan\theta = 0.2446$ (slope downward)

$\therefore$ $\theta = \tan^{-1} 0.2446 = 13.7446°$ or **13° 44.6′. Ans.**

(*ii*) **The maximum and minimum pressure intensities at the bottom of the tank**

From the Fig. 3.45,

Depth of water at the front end,

$$h_1 = 1 - 3\tan\theta = 1 - 3 \times 0.2446 = 0.2662 \text{ m}$$

Depth of water at the rear end,

$$h_2 = 1 + 3\tan\theta = 1 + 3 \times 0.2446 = 1.7338 \text{ m}$$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now the maximum pressure intensity at the bottom will be at point *A* and it is given by,

$$p_{max} = \rho \times g \times h_2$$
$$= 1000 \times 9.81 \times 1.7338 \text{ N/m}^2 = \mathbf{17008.5\ N/m^2.\ Ans.}$$

The minimum pressure intensity at the bottom will be at point B and it is given by

$$p_{min} = \rho \times g \times h_1$$
$$= 1000 \times 9.81 \times 0.2662 = \mathbf{2611.4\ N/m^2.\ Ans.}$$

(*iii*) **The total force due to water acting on each end of the tank**

Let F_1 = total force acting on the front side (*i.e.*, on face *BD*)

F_2 = total force acting on the rear side (*i.e.*, on face *AC*)

Then $F_1 = \rho g A_1 \bar{h}_1$, where $A_1 = BD \times$ width of tank $= h_1 \times 2.5 = 0.2662 \times 2.5$

and $\bar{h}_1 = \frac{BD}{2} = \frac{h_1}{2} = \frac{0.2662}{2} = 0.1331$ m

$$= 1000 \times 9.81 \times (0.2662 \times 2.5) \times 0.1331$$
$$= \mathbf{868.95\ N.\ Ans.}$$

Hence difference between the forces on the two ends of the tank is equal to the force necessary to accelerate the mass of water in the tank.

(*iii*) (*a*) *Horizontal acceleration when the bottom of the tank is exposed upto its mid-point.*

Refer to Fig. 3.47 (*c*). In this case the free surface of water in the tank will be along *CD**, where *D** is the mid-point of *ED*.

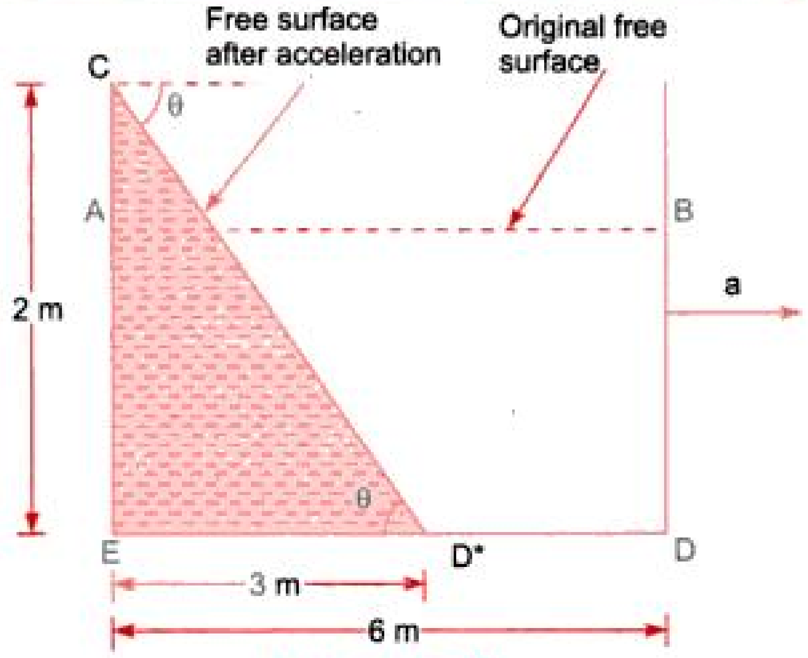

Fig. 3.47 (*c*)

Let a = required horizontal acceleration from Fig. 3.47 (*c*), it is clear that

$$\tan\theta = \frac{CE}{ED} = \frac{2}{3}$$

But from equation (3.20) numerically

$$\tan\theta = \frac{a}{g}$$

$$\therefore \qquad a = g \times \tan\theta = 9.81 \times \frac{2}{3} = \mathbf{6.54\ m/s^2.\ Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end *CE* of the tank is

$$F_1 = \rho \times g \times A_1 \times \bar{h}_1$$

where $A_1 = CE \times \text{Width} = 2 \times 2.5 = 5\ \text{m}^2$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1\ \text{m}$$

$$= 1000 \times 9.81 \times 5 \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end *BD* is zero as there is no water against the face *BD*.

$$\therefore \qquad F_2 = 0$$

$\therefore$ Difference of the forces $= F_1 - F_2 = 49050 - 0 = 49050$ M

(*c*) *Difference of the two forces is equal to the force necessary to accelerate the mass of water remaining in the tank.*

Volume of water in the tank = Area *CED* × Width of tank

$$= \frac{CE \times ED}{2} \times 2.5 = \frac{2 \times 3}{2} \times 2.5 = 7.5\ \text{m}^3$$

Force necessary to accelerate the mass of water in the tank

= Mass of water × Acceleration

$= \rho \times$ Volume of water $\times 6.54$ $\qquad (\because a = 6.54\ \text{m/s}^2)$

$= 1000 \times 7.5 \times 6.54$

$= 49050$ N

This is the same force as the difference of the two forces on the two ends of the tank.

Problem 3.36 *A rectangular tank of length 6 m, width 2.5 and height 2 m is completely filled with water when at rest. The tank is open at the top. The tank is subjected to a horizontal constant linear acceleration of 2.4 m/s² in the direction of its length. Find the volume of water spilled from the tank.*

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(*ii*) **Force on the side of the tank, when tank is stationary.**

The pressure at point B is given by,

$$p_B = \rho g h = 1000 \times 9.81 \times 0.5 = 4905 \text{ N/m}^2$$

This pressure is represented by line BD in Fig. 3.52

Force on the side AB $= \text{Area of triangle } ABD \times \text{Width}$

$$= \left(\tfrac{1}{2} \times AB \times BD\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 4905\right) \times 2 \qquad (\because BD = 4905)$$

$$= \mathbf{2452.5 \text{ N. Ans.}}$$

For this case, the force on AB can also be obtained as

$$F_{AB} = \rho g A.\bar{h}$$

where $A = AB \times \text{Width} = 0.5 \times 2 = 1 \text{ m}^2$

$$\bar{h} = \frac{AB}{2} = \frac{0.5}{2} = 0.25 \text{ m} = 1000 \times 9.81 \times 1 \times 0.25$$

$$= \mathbf{2452.5 \text{ N. Ans.}}$$

Problem 3.38 *A tank contains water upto a depth of 1.5 m. The length and width of the tank are 4 m and 2 m respectively. The tank is moving up an inclined plane with a constant acceleration of 4 m/s². The inclination of the plane with the horizontal is 30° as shown in Fig. 3.53. Find,*

(i) the angle made by the free surface of water with the horizontal.

(ii) the pressure at the bottom of the tank at the front and rear ends.

Solution. Given :

Depth of water, $h = 1.5$ m ; Length, $L = 4$ m and Width, $b = 2$ m

Constant acceleration along the inclined plane,

$$a = 4 \text{ m/s}^2$$

Inclination of plane, $\alpha = 30°$

Let θ = Angle made by the free surface of water after the acceleration is imparted to the tank

p_A = Pressure at the bottom of the tank at the front end and

p_D = Pressure at the bottom of the tank at the rear end.

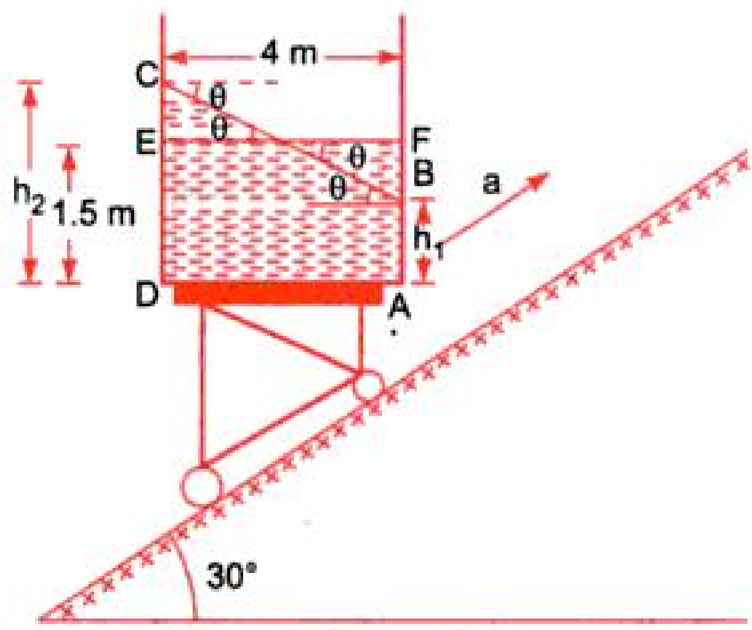

Fig. 3.53

This problem can be done by resolving the given acceleration along the horizontal direction and vertical directions. Then each of these cases may be separately analysed according to the set procedure.

Horizontal and vertical components of the acceleration are :

$$a_x = a \cos \alpha = 4 \cos 30° = 3.464 \text{ m/s}^2$$

$$a_y = a \sin \alpha = 4 \sin 30° = 2 \text{ m/s}^2$$

When the tank is stationary on the inclined plane, free surface of liquid will be along EF as shown in Fig. 3.53. But when the tank is moving upward along the inclined plane the free surface of liquid will be along BC. When the tank containing a liquid is moving up an inclined plan with a constant acceleration, the angle made by the free surface of the liquid with the horizontal is given by

8. A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 12 m wide at the bottom and 8 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is 1 m below the top level of the caisson and dock is empty. *(A.M.I.E., Winter 1980)*
[**Ans.** 3.164 MN, 4.56 m below water surface]

9. A sliding gate 2 m wide and 1.5 m high lies in a vertical plane and has a co-efficient of friction of 0.2 between itself and guides. If the gate weighs one tonne, find the vertical force required to raise the gate if its upper edge is at a depth of 4 m from free surface of water. [**Ans.** 37768.5 N]

10. A tank contains water upto a height of 1 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1.5 m height. Calculate : (*i*) total pressure on one side of the tank, (*ii*) the position of centre of pressure for one side of the tank, which is 3 m wide. [**Ans.** 76518 N, 1.686 m from top]

11. A rectangular tank 4 m long, 1.5 m wide contains water upto a height of 2 m. Calculate the force due to water pressure on the base of the tank. Find also the depth of centre of pressure from free surface.
[**Ans.** 117720 N, 2 m from free surface]

12. A rectangular plane surface 1 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface. [**Ans.** 80932.5 N, 2.318 m]

13. A circular plate 3.0 m diameter is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface.
[**Ans.** 228.69 kN, 3.427 m from free surface]

14. A rectangular gate 6 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.54. To keep the gate in a stable position, a counter weight of 29430 N is attached at the upper end of the gate. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and also friction at the hinge and pulley. [**Ans.** 3.43 m]

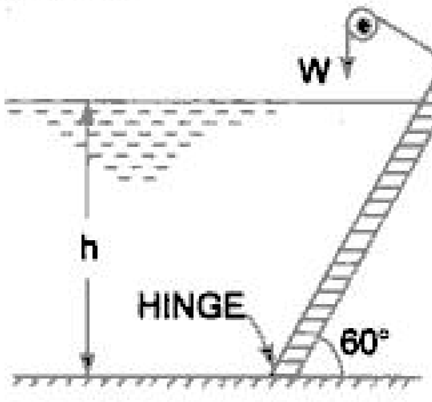

Fig. 3.54

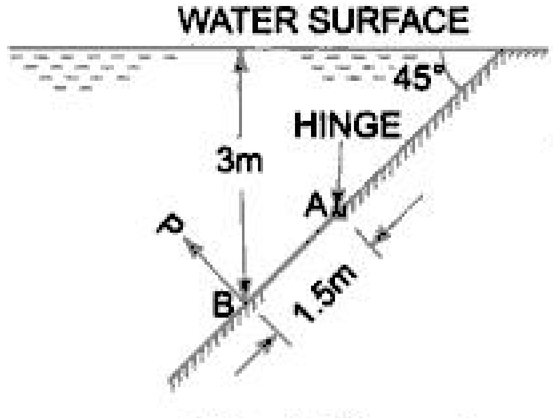

Fig. 3.55

15. An inclined rectangular gate of width 5 m and depth 1.5 m is installed to control the discharge of water as shown in Fig. 3.55. The end *A* is hinged. Determine the force normal to the gate applied at *B* to open it.
[**Ans.** 97435.8 N]

16. A gate supporting water is shown in Fig. 3.56. Find the height '*h*' of the water so that the gate begins to tip about the hinge. Take the width of the gate as unity. *(Delhi University, 1986)*
[**Ans.** $3 \times \sqrt{3}$ m]

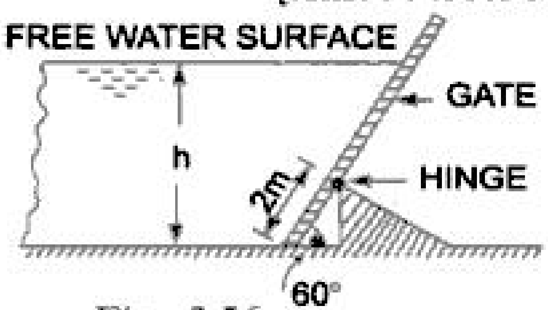

Fig. 3.56

17. Find the total pressure and depth of centre of pressure on a triangular plate of base 3 m and height 3 m which is immersed in water in such a way that plane of the plate makes an angle of 60° with the free surface. The base of the plate is parallel to water surface and at a depth of 2 m from water surface. [**Ans.** 126.52 kN, 2.996 m]

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For equilibrium the weight of water displaced = Weight of wooden block

$= 143471$ N

$\therefore$ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625\ m^3.\ Ans.}$$

($\because$ Weight density of water = 1000×9.81 N/m^3)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

or $2.5 \times h \times 6.0 = 14.625$ m^3, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875\ m\ from\ base.\ Ans.}$$

Problem 4.2 *A wooden log of 0.6 m diameter and 5 m length is floating in river water. Find the depth of the wooden log in water when the sp. gravity of the log is 0.7.*

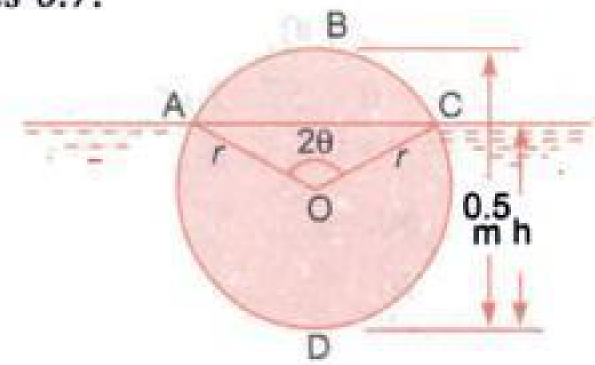

Fig. 4.2

Solution. Given :

Dia. of log $= 0.6$ m

Length, $L = 5$ m

Sp. gr., $S = 0.7$

$\therefore$ Density of log $= 0.7 \times 1000 = 700$ kg/m^3

$\therefore$ Weight density of log, $w = \rho \times g$

$= 700 \times 9.81$ N/m^3

Find depth of immersion or h

Weight of wooden log = Weight density × Volume of log

$$= 700 \times 9.81 \times \frac{\pi}{4}(D)^2 \times L$$

$$= 700 \times 9.81 \times \frac{\pi}{4}(.6)^2 \times 5 \text{ N} = 989.6 \times 9.81 \text{ N}$$

For equilibrium,

Weight of wooden log = Weight of water displaced

= Weight density of water × Volume of water displaced

$$\therefore \quad \text{Volume of water displaced} = \frac{989.6 \times 9.81}{1000 \times 9.81} = 0.9896 \text{ m}^3$$

($\because$ Weight density of water = 1000×9.81 N/m^3)

Let h is the depth of immersion

$\therefore$ Volume of log inside water = Area of $ADCA$ × Length

= Area of $ADCA$ × 5.0

But volume of log inside water = Volume of water displaced = 0.9896 m^3

$= \frac{2}{3} \times \pi \times (.075)^3 + .00355 \times \pi \times (.075)^2 = 0.000945 \text{ m}^3$

$\therefore$ Buoyant force = Weight of oil displaced

$= \rho_0 \times g \times$ Volume of oil

$= 800 \times 9.81 \times .000945 = 7.416$ N

The buoyant force and weight of the float passes through the same vertical line, passing through B. Let the weight of float is W. Then net vertical force on float

$= \text{Buoyant force} - \text{Weight of float} = (7.416 - W)$

Taking moments about the hinge O, we get

$$P \times 20 = (7.416 - W) \times BD = (7.416 - W) \times 50 \times \cos 45°$$

or
$$9.81 \times 20 = (7.416 - W) \times 35.355$$

$\therefore$
$$W = 7.416 - \frac{20 \times 9.81}{35.355} = 7.416 - 5.55 = \mathbf{1.866 \text{ N. Ans.}}$$

▶ 4.4 META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

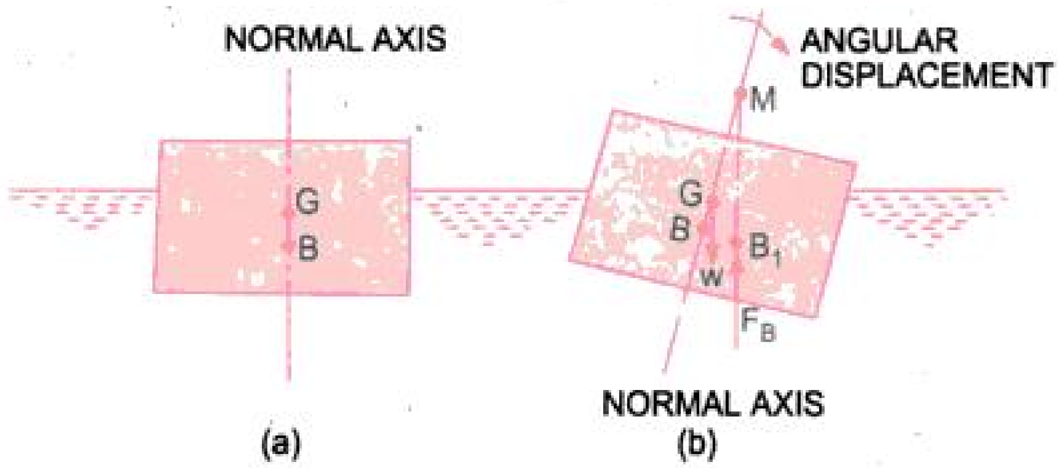

Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

▶ 4.5 META-CENTRIC HEIGHT

The distance MG, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

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Problem 4.9 *A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m × 1 m × 0.8 m.*

Solution. Given :

Dimension of block $= 2 \times 1 \times 0.8$

Let depth of immersion $= h$ m

Sp. gr. of wood $= 0.7$

Weight of wooden piece = Weight density of wood* × Volume

$= 0.7 \times 1000 \times 9.81 \times 2 \times 1 \times 0.8$ N

Weight of water displaced = Weight density of water × Volume of the wood sub-merged in water

$= 1000 \times 9.81 \times 2 \times 1 \times h$ N

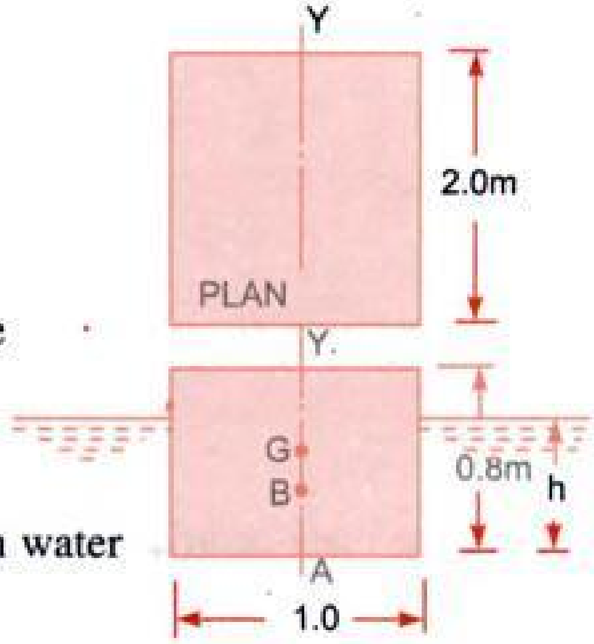


Fig. 4.9

For equilibrium,

Weight of wooden piece = Weight of water displaced

$\therefore \quad 700 \times 9.81 \times 2 \times 1 \times 0.8 = 1000 \times 9.81 \times 2 \times 1 \times h$

$$\therefore \quad h = \frac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1} = 0.7 \times 0.8 = 0.56 \text{ m}$$

$\therefore$ Distance of centre of Buoyancy from bottom, *i.e.*,

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 \text{ m}$$

and $\quad AG = 0.8/2.0 = 0.4$ m

$\therefore \quad BG = AG - AB = 0.4 - 0.28 = 0.12$ m

The meta-centric height is given by equation (4.4) or

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{1}{12} \times 2 \times 1.0^3 = \frac{1}{6}$ m^4

$\forall$ = Volume of wood in water

$= 2 \times 1 \times h = 2 \times 1 \times .56 = 1.12$ m^3

$$\therefore \quad GM = \frac{1}{6} \times \frac{1}{1.12} - 0.12 = 0.1488 - 0.12 = \mathbf{0.0288 \text{ m. Ans.}}$$

Problem 4.10 *A solid cylinder of diameter 4.0 m has a height of 3 metres. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder = 0.6.*

Solution. Given :

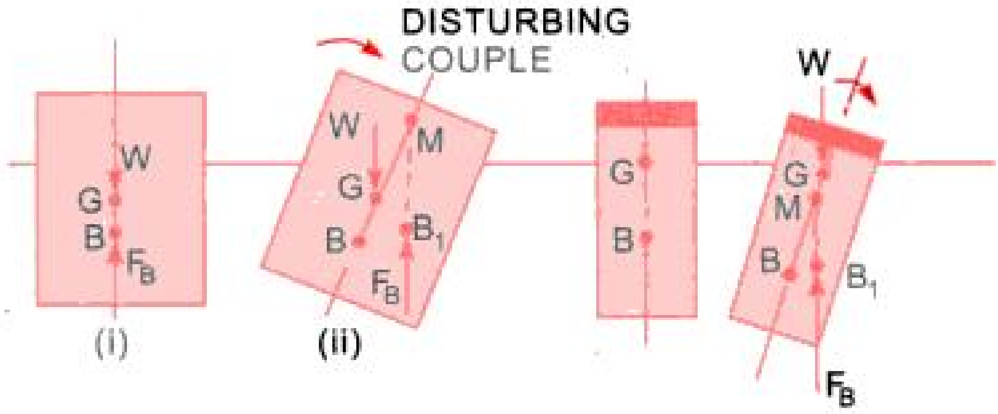
Dia. of cylinder, $\quad D = 4.0$ m

Height of cylinder, $\quad h = 3.0$ m

---

* Weight density of wood $= \rho \times g$, where ρ = density of wood $= 0.7 \times 1000 = 700$ kg/m^3. Hence w for wood $= 700 \times 9.81$ N/m^3.

(*a*) **Stable Equilibrium.** If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(*a*) Stable equilibrium M is above G (*b*) Unstable equilibrium M is below G.

Fig. 4.13 *Stability of floating bodies.*

(*b*) **Unstable Equilibrium.** If the point M is below G, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Problem 4.12 *A solid cylinder of diameter 4.0 m has a height of 4.0 m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.*

Solution. Given : $D = 4$ m

Height, $h = 4$ m

Sp. gr. $= 0.6$

Depth of cylinder in water $=$ Sp. gr. $\times h$

$= 0.6 \times 4.0 = 2.4$ m

$\therefore$ Distance of centre of buoyancy (B) from A

or $$AB = \frac{2.4}{2} = 1.2 \text{ m}$$

Distance of centre of gravity (G) from A

or $$AG = \frac{h}{2} = \frac{4.0}{2} = 2.0 \text{ m}$$

$\therefore$ $$BG = AG - AB = 2.0 - 1.2 = 0.8 \text{ m}$$

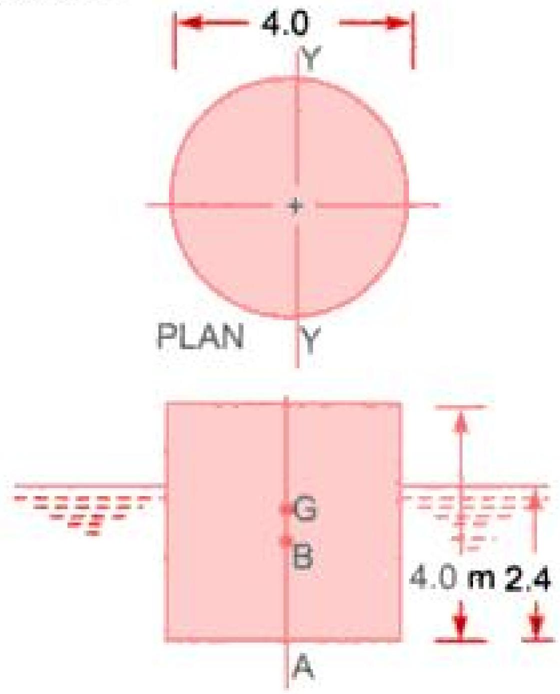


Fig. 4.14

Now the meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

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Sp. gr. of oil $S_2 = 0.9$

Let the depth of cylinder immersed in oil = h

For the principle of buoyancy

Weight of cylinder = wt. of oil displaced

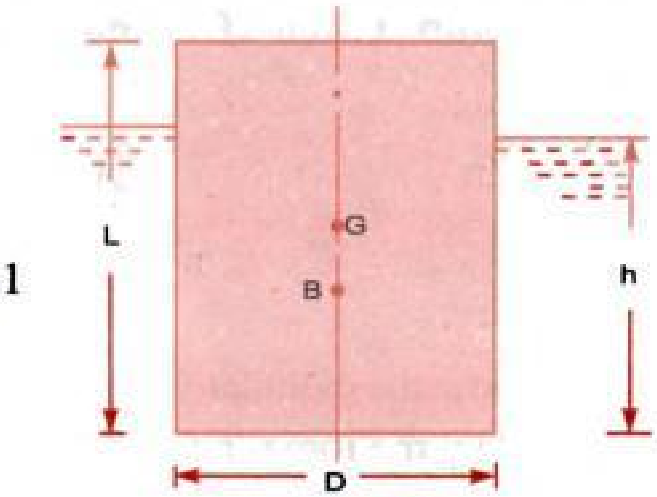

Fig. 4.18

$$\frac{\pi}{4} D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4} D^2 \times h \times 0.9 \times 1000 \times 9.81$$

or $$L \times 0.6 = h \times 0.9$$

$\therefore$ $$h = \frac{0.6 \times L}{0.9} = \frac{2}{3} L.$$

The distance of centre of gravity C from A, $AG = \frac{L}{2}$

The distance of centre of buoyancy B from A,

$$AB = \frac{h}{2} = \frac{1}{2}\left[\frac{2}{3}L\right] = \frac{L}{3}$$

$\therefore$ $$BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{6}$$

The meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{\pi}{64} D^4$ and $\forall$ = volume of cylinder in oil = $\frac{\pi}{4} D^2 \times h$

$\therefore$ $$\frac{I}{\forall} = \left(\frac{\pi}{64} D^4 \Big/ \frac{\pi}{4} D^2 h\right) = \frac{1}{16}\frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L} \qquad \left\{\because\ h = \frac{2}{3} L\right\}$$

$\therefore$ $$GM = \frac{3D^3}{32L} - \frac{L}{6}.$$

For stable equilibrium, GM should be +ve or

$$GM > 0 \quad \text{or} \quad \frac{3D^2}{32L} - \frac{L}{6} > 0$$

or $$\frac{3D^2}{32L} > \frac{L}{6} \quad \text{or} \quad \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

or $$\frac{L^2}{D^2} < \frac{18}{32} \quad \text{or} \quad \frac{9}{16}$$

$\therefore$ $$\frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore$ **L/D < 3/4. Ans.**

Problem 4.16 *Show that a cylindrical buoy of 1 m diameter and 2.0 m height weighing 7.848 kN will not float vertically in sea water of density 1030 kg/m³. Find the force necessary in a vertical chain attached at the centre of base of the buoy that will keep it vertical.*

$\therefore$ Weight of cone $= 800 \times g \times \frac{1}{3} \times \pi \times (H \tan \theta)^2 \times H = \dfrac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3}$

$\therefore$ Weight of water displaced $= 1000 \times g \times \frac{1}{3} \times \pi r^2 \times h$

$$= 1000 \times g \times \tfrac{1}{3} \times \pi (h \tan \theta)^2 \times h = \frac{1000 \times g \times \pi \times h^3 \tan^2 \theta}{3.0}$$

For equilibrium

Weight of cone = Weight of water displaced

or $$\frac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3.0} = \frac{1000 \times 9.81 \times \pi \times h^3 \times \tan^2 \theta}{3.0}$$

or $$800 \times H^3 = 1000 \times h^3$$

$\therefore$ $$H^3 = \frac{1000}{800} \times h^3 \text{ or } \frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3}$$

For stable equilibrium, Meta-centric height GM should be positive. But GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of cone at water-line $= \dfrac{\pi}{64} d^4$

$\forall$ = Volume of cone in water $= \dfrac{1}{3}\dfrac{\pi}{4} d^2 \times h$

$\therefore$ $$\frac{I}{\forall} = \frac{\pi}{64} d^4 \Big/ \frac{1}{3} \times \frac{\pi}{4} d^2 \times h$$

$$= \frac{1 \times 3}{16} \times \frac{d^2}{h} = \frac{3d^2}{16h} = \frac{3}{16h} \times (2r)^2 = \frac{3}{4}\frac{r^2}{h}$$

$$= \frac{3}{4}\frac{(h \tan \theta)^2}{h} \qquad \{\because \quad r = h \tan \theta\}$$

$$= \tfrac{3}{4} h \tan^2 \theta$$

and $$BG = AG - AB = \tfrac{3}{4} H - \tfrac{3}{4} h = \tfrac{3}{4}(H - h)$$

$\therefore$ $$GM = \tfrac{3}{4} h \tan^2 \theta - \tfrac{3}{4}(H - h)$$

For stable equilibrium GM should be positive or

$$\tfrac{3}{4} h \tan^2 - \tfrac{3}{4}(H - h) > 0 \quad \text{or} \quad h \tan^2 \theta - (H - h) > 0$$

or $$h \tan^2 \theta > (H - h) \quad \text{or} \quad h \tan^2 \theta + h > \text{H}$$

or $$h[\tan^2 \theta + 1] > H \quad \text{or} \quad 1 + \tan^2 \theta > H/h \quad \text{or} \quad \sec^2 \theta > \frac{H}{h}$$

But $$\frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3} = 1.077$$

$\therefore$ $$\sec^2 \theta > 1.077 \text{ or } \cos^2 \theta > \frac{1}{1.077} = 0.9285$$

$\therefore$ $$\cos \theta > 0.9635$$

$\therefore$ $$\theta > 15° \, 30' \quad \text{or} \quad 2\theta > 31°$$

$\therefore$ Apex angle (2θ) should be at least **31°. Ans.**

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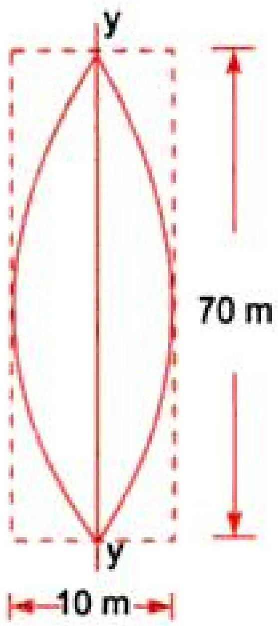

Fig. 4.24

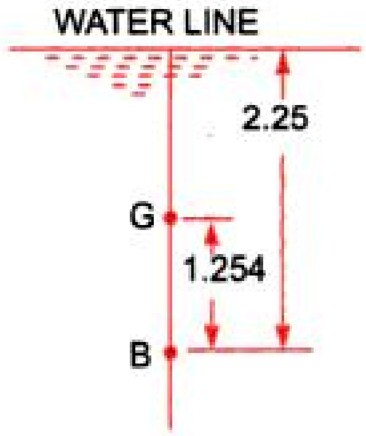

Fig. 4.25

$$= 75\% \text{ of } \frac{1}{12} \times 70 \times 10^3 = .75 \times \frac{1}{12} \times 70 \times 10^3 = 4375 \text{ m}^4$$

and $\quad \forall$ = Volume of ship in water $= \dfrac{\text{Weight of ship}}{\text{Weight densit of water}} = \dfrac{19620}{10.104} = 1941.74 \text{ m}^3$

$$\therefore \quad \frac{I}{\forall} = \frac{4375}{1941.74} = 2.253 \text{ m}$$

$$\therefore \quad GM = 2.253 - BG \text{ or } .999 = 2.253 - BG$$

$$\therefore \quad BG = 2.253 - .999 = 1.254 \text{ m.}$$

From Fig. 4.25, it is clear that the distance of G from free surface of the water = distance of B from water surface $-$ BG

$$= 2.25 - 1.254 = \mathbf{0.996 \text{ m. Ans.}}$$

Problem 4.20 *A pontoon of 15696 kN displacement is floating in water. A weight of 245.25 kN is moved through a distance of 8 m across the deck of pontoon, which tilts the pontoon through an angle 4°. Find meta-centric height of the pontoon.*

Solution. Given :

Weight of pontoon = Displacement

or $\quad W = 15696$ kN

Movable weight, $\quad w_1 = 245.25$ kN

Distance moved by weight w_1, $x = 8$ m

Angle of heel, $\quad \theta = 4°$

The meta-centric height, GM is given by equation (4.5)

or
$$GM = \frac{w_1 x}{W \tan\theta} = \frac{245.25 \text{ kN} \times 8}{15696 \text{ kN} \times \tan 4°}$$

$$= \frac{1962}{15696 \times 0.0699} = \mathbf{1.788 \text{ m. Ans.}}$$

▶ 4.9 OSCILLATION (ROLLING) OF A FLOATING BODY

Consider a floating body, which is tilted through an angle by an overturning couple as shown in Fig. 4.26. Let the over-turning couple is suddenly removed. The body will start oscillating. Thus, the

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EXERCISE 4

(A) THEORETICAL PROBLEMS

1. Define the terms 'buoyancy' and 'centre of buoyancy'.
2. Explain the terms 'meta-centre' and 'meta-centric height'.
3. Derive an expression for the meta-centric height of a floating body.
4. Show that the distance between the meta-centre and centre of buoyancy is given by $BM = \frac{I}{\forall}$

   where I = Moment of inertia of the plan of the floating body at water surface about longitudinal axis.

   $\forall$ = Volume of the body submerged in liquid.
5. What are the conditions of equilibrium of a floating body and a submerged body ? *(A.S.M.E., June 1992 ; Delhi University, 1982)*
6. How will you determine the meta-centric height of a floating body experimentally ? Explain with neat sketch.
7. Select the correct statement :

   (*a*) The buoyant force for a floating body passes through the

   (*i*) centre of gravity of the body (*ii*) centroid of volume of the body

   (*iii*) meta-centre of the body (*iv*) centre of gravity of the submerged part of the body

   (*v*) centroid of the displaced volume.

   (*b*) A body submerged in liquid is in equilibrium when :

   (*i*) its meta-centre is above the centre of gravity

   (*ii*) its meta-centre is above the centre of buoyancy

   (*iii*) its centre of gravity is above the centre of buoyancy

   (*iv*) its centre of buoyancy is above the centre of gravity

   (*v*) none of these. [**Ans.** 7 (*a*) (*v*), (*b*) (*iv*)]
8. Derive an expression for the time period of the oscillation of a floating body in terms of radius of gyration and meta-centric height of the floating body.
9. Define the terms : meta-centre, centre of buoyancy, meta-centric height, gauge pressure and absolute pressure. *(A.S.M.E., June 1992)*
10. What do you understand by the hydrostatic equation ? With the help of this equation, derive the expression for the buoyant force acting on a sub-merged body. *(A.M.I.E. S 1990)*
11. With neat sketches, explain the conditions of equilibrium for floating and sub-merged bodies. *(Delhi University, June 1996)*
12. Differentiate between :

    (*i*) Dynamic viscosity and kinematic viscoscity, (*ii*) Absolute and gauge pressure (*iii*) Simple and differential manometers (*iv*) Centre of gravity and centre of buoyancy. *(Delhi University, Dec. 2002)*

(B) NUMERICAL PROBLEMS

1. A wooden block of width 2 m, depth 1.5 m and length 4 m floats horizontally in water. Find the volume of water displaced and position of centre buoyancy. The specific gravity of the wooden block is 0.7. [**Ans.** 8.4 m^3, 0.525 m from the base]

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure of density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

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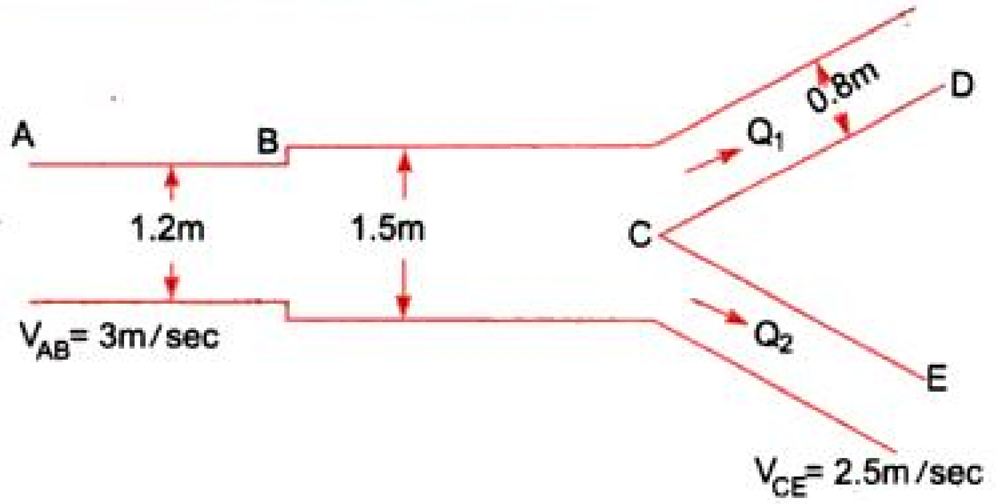

Fig. 5.4

Diameter of pipe $CE = D_{CE}$

Then flow rate through $CD = Q/3$

and flow rate through $CE = Q - Q/3 = \frac{2Q}{3}$

(*i*) Now volume flow rate through $AB = Q = V_{AB} \times$ Area of AB

$$= 3.0 \times \frac{\pi}{4} (D_{AB})^2 = 3.0 \times \frac{\pi}{4} (1.2)^2 = \textbf{3.393 m}^3\textbf{/s. Ans.}$$

(*ii*) Applying continuity equation to pipe AB and pipe BC,

$$V_{AB} \times \text{Area of pipe } AB = V_{BC} \times \text{Area of pipe } BC$$

or $$3.0 \times \frac{\pi}{4} (D_{AB})^2 = V_{BC} \times \frac{\pi}{4} (D_{BC})^2$$

or $$3.0 \times (1.2)^2 = V_{BC} \times (1.5)^2 \qquad \left[\text{Divide by } \frac{\pi}{4}\right]$$

or $$V_{BC} = \frac{3 \times 1.2^2}{1.5^2} = \textbf{1.92 m/s. Ans.}$$

(*iii*) The flow rate through pipe

$$C_D = Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131 \text{ m}^3\text{/s}$$

∴ $$Q_1 = V_{CD} \times \text{Area of pipe } C_D \times \frac{\pi}{4} (C_{CD})^2$$

or $$1.131 = V_{CD} \times \frac{\pi}{4} \times .8^2 = 0.5026 \ V_{CD}$$

∴ $$V_{CD} = \frac{1.131}{0.5026} = \textbf{2.25 m/s. Ans.}$$

(*iv*) Flow rate through CE,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \text{ m}^3\text{/s}$$

∴ $$Q_2 = V_{CE} \times \text{Area of pipe } CE = V_{CE} \frac{\pi}{4} (D_{CE})^2$$

or $$2.263 = 2.5 \times \frac{\pi}{4} \times (D_{CE})^2$$

or $$D_{CE} = \sqrt{\frac{2.263 \times 4}{2.5 \times \pi}} = \sqrt{1.152} = 1.0735 \text{ m}$$

∴ Diameter of pipe CE = **1.0735 m. Ans.**

$$= \rho \times u_r \times (AB \times 1) \qquad (\because \text{ Area} = AB \times \text{Thickness} = rd\theta \times 1)$$

$$= \rho \times u_r \times (rd\theta \times 1) = \rho \, . \, u_r . \, rd\theta$$

Mass of fluid leaving the face CD per unit time

$$= \rho \times \text{Velocity} \times \text{Area}$$

$$= \rho \times \left(u_r + \frac{\partial u_r}{\partial r} . dr \right) \times (CD \times 1) \qquad (\because \text{ Area} = CD \times 1)$$

$$= \rho \times \left(u_r + \frac{\partial u_r}{\partial r} dr \right) \times (r + dr) d\theta \qquad [\because CD = (r + dr)\, d\theta]$$

$$= \rho \times \left[u_r \times r + u_r dr + r \frac{\partial u_r}{\partial r} dr + \frac{\partial u_r}{\partial r} (dr)^2 \right] d\theta$$

$$= \rho \left[u_r \times r + u_r \times dr + r \frac{\partial u_r}{\partial r} . dr \right] d\theta$$

[The term containing $(dr)^2$ is very small and has been neglected]

$\therefore$ Gain of mass in r-direction per unit time

$$= (\text{Mass through } AB - \text{Mass through } CD) \text{ per unit time}$$

$$= \rho . \, u_r . \, rd\theta - \rho \left[u_r . r + u_r . dr + r \frac{\partial u_r}{\partial r} . dr \right] d\theta$$

$$= \rho . \, u_r . \, rd\theta - \rho . \, u_r . \, r . \, d\theta - \rho \left[u_r . dr + r \frac{\partial u_r}{\partial r} . dr \right] d\theta$$

$$= - \rho \left[u_r . dr + r \frac{\partial u_r}{\partial r} . dr \right] . \, d\theta$$

$$= - \rho \left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right] r . \, dr . \, d\theta$$

[This is written in this form because (r. $d\theta$. dr. 1) is equal to volume of element]

Now consider the flow in θ-direction

Gain in mass in θ-direction per unit time

$$= (\text{Mass through } BC - \text{Mass through } AD) \text{ per unit time}$$

$$= [\rho \times \text{Velocity through } BC \times \text{Area} - \rho \times \text{Velocity through } AD \times \text{Area}]$$

$$= \left[\rho . u_\theta . dr \times 1 - \rho \left(u_\theta + \frac{\partial u_\theta}{\partial \theta} . d\theta \right) \times dr \times 1 \right]$$

$$= - \rho \left(\frac{\partial u_\theta}{\partial \theta} . d\theta \right) dr \times 1 \qquad (\because \text{ Area} = dr \times 1)$$

$$= - \rho \frac{\partial u_\theta}{\partial \theta} . \frac{r \, . \, d\theta \, . \, dr}{r} \qquad \text{[Multiplying and dividing by } r\text{]}$$

$\therefore$ Total gain in fluid mass per unit time

$$= - \rho \left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right] . \, r . \, dr . \, d\theta - \rho \frac{\partial u_\theta}{\partial \theta} . \frac{rd\theta \, . \, dr}{r} \qquad ...(5.5A)$$

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$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2, \frac{\partial u}{\partial y} = 0, \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy, \frac{\partial v}{\partial y} = -10x^2, \frac{\partial v}{\partial z} = 0\ \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial y} = 0, \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.0$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t \times (0) + 0$$
$$= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units}$$
$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$
$$= -80x^4y + 100x^4y$$
$$= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.}$$
$$a_z = 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.0 = 2.0 \text{ units}$$

$\therefore$ Acceleration is $A = a_x i + a_y j + a_z k = \mathbf{1536i + 320j + 2k. \ Ans.}$

or Resultant $A = \sqrt{(1536)^2 + (320)^2 + (2)^2}$ units

$$= \sqrt{2359296 + 102400 + 4} = \mathbf{1568.9 \text{ units. Ans.}}$$

Problem 5.7 *The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation :*

(i) $u = x^2 + y^2 + z^2$; $v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$, $w = 2xyz$.

Solution. The continuity equation for incompressible fluid is given by equation (5.4) as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Case I. $u = x^2 + y^2 + z^2 \quad \therefore \quad \frac{\partial u}{\partial x} = 2x$

$v = xy^2 - yz^2 + xy \quad \therefore \quad \frac{\partial v}{\partial y} = 2xy - z^2 + x$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation.

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

or
$$\frac{\partial w}{\partial z} = -3x - 2xy + z^2 \text{ or } \partial w = (-3x - 2xy + z^2)\,\partial z$$

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$$= Q_1 + \frac{Q_2 - Q_1}{30} \times 15 \qquad \text{where } Q_1 = 20 \text{ lit/s, } Q_2 = 40 \text{ lit/s}$$

$$= 20 + \frac{40-20}{30} \times 15 = 20 + \frac{20}{30} \times 15 = 30 \text{ lit/s} = 30000 \text{ cm}^3\text{/s}$$

$$\therefore \qquad u_1 = \frac{Q}{A_1} = \frac{30000}{\frac{\pi}{4}(40)^2} = \frac{30000}{1256.63} = 23.85 \text{ cm/s}$$

$$u_2 = \frac{Q}{A_2} = \frac{30000}{\frac{\pi}{4}(20)^2} = 95.40 \text{ cm/s}$$

$\therefore$ The velocity at $$A = u_1 + \frac{u_2 - u_1}{L} \times x$$

or $$u = 23.85 + \frac{95.40 - 23.85}{200} \times x = 23.85 + .35775\, x$$

and $$\frac{\partial u}{\partial x} = .35775$$

$$\therefore \qquad \mu \frac{\partial u}{\partial x} = (23.85 + .35775x)\ .35775$$

$\therefore$ Convective acceleration at $x = 100$

$$= (23.85 + .35775 \times 100\ ) \times .35775 \text{ cm/s}^2$$
$$= (23.85 + 35.775) \times .35775 = 21.33 \text{ cm}^2\text{/s.}$$

For Local Acceleration. Find rate of change of velocity at point A.

The diameter at $$A = D_1 + \frac{(D_2 - D_1)}{L} \times 100$$

$$= 40 + \frac{20-40}{200} \times 100 = 40 - 10 = 30 \text{ cm.}$$

Find velocity at A, when $Q_1 = 20$ lit/s and $Q_2 = 40$ lit/s

$\therefore$ When $$Q = 20 \text{ lit/s, } u = \frac{Q}{\text{Area}} = \frac{20000}{\frac{\pi}{4}(30)^2} = 28.29 \text{ cm/s}$$

When $$Q = 40 \text{ lit/s, } u = \frac{40000}{\frac{\pi}{4}(30)^2} = 56.58 \text{ cm/s}$$

$$\therefore \text{ Rate of change of velocity} = \frac{\text{Change of velocity}}{\text{Time}} = \frac{56.28 - 28.29}{30}$$

or Local acceleration $$= \frac{28.29}{30} = 0.943 \text{ cm/s}^2$$

$\therefore$ Total acceleration $= 21.33 + .943 = 22.273$ cm/s^2 = **22.273 cm/s^2. Ans.**

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For a line of constant stream function

$$= d\psi = 0 \text{ or } vdx - udy = 0$$

or
$$\frac{dy}{dx} = \frac{v}{u} \qquad ...(5.14)$$

But $\dfrac{dy}{dx}$ is slope of stream line.

From equations (5.13) and (5.14) it is clear that the product of the slope of the equipotential line and the slope of the stream line at the point of intersection is equal to – 1. Thus the equipotential lines are othogonal to the stream lines at all points of intersection.

5.8.5 Flow Net. A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net. The flow net is an important tool in analysing two-dimensional irrotational flow problems.

5.8.6 Relation between Stream Function and Velocity Potential Function, From equation (5.9),

we have
$$u = -\frac{\partial\phi}{\partial x} \text{ and } v = -\frac{\partial\phi}{\partial y}$$

From equation (5.12), we have $u = -\dfrac{\partial\psi}{\partial y}$ and $v = \dfrac{\partial\psi}{\partial x}$

Thus, we have
$$u = -\frac{\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$$

Hence
$$\left.\begin{aligned} \frac{\partial\phi}{\partial x} &= \frac{\partial\psi}{\partial y} \\ \text{and} \quad \frac{\partial\phi}{\partial y} &= -\frac{\partial\psi}{\partial x} \end{aligned}\right\} \qquad ...(5.15)$$

Problem 5.10 *The velocity potential function* (ϕ) *is given by an expression*

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3y}{3} + y^2$$

(*i*) *Find the velocity components in x and y direction.*
(*ii*) *Show that* ϕ *represents a possible case of flow.*

Solution. Given :
$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3y}{3} + y^2$$

The partial derivatives of ϕ w.r.t. to x and y are

$$\frac{\partial\phi}{\partial x} = -\frac{y^3}{3} - 2x + \frac{3x^2y}{3} \qquad ...(1)$$

and
$$\frac{\partial\phi}{\partial y} = -\frac{3xy^2}{3} + \frac{x^3}{3} + 2y \qquad ...(2)$$

(*i*) The velocity components u and v are given by equation (5.9)

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But from (*ii*), $$\frac{\partial \phi}{\partial y} = -2y$$

$$\therefore \qquad \frac{\partial C}{\partial y} = -2y$$

Integrating this equation, we get $C = \int -2y\, dy = -\dfrac{2y^2}{2} = -y^2$

Substituting this value of C in equation (*iii*), we get $\phi = \mathbf{x^2 - y^2}$. **Ans.**

Problem 5.15 *Sketch the stream lines represented by $y = x^2 + y^2$. Also find out the velocity and its direction at point (1, 2).* (A.M.I.E., Winter, 1980)

Solution. Given : $\psi = x^2 + y^2$

The velocity components u and v are

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 + y^2) = -2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2) = 2x$$

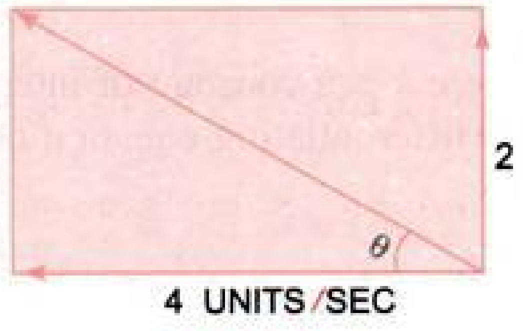

Fig. 5.9

At the point (1, 2), the velocity components are

$$u = -2 \times 2 = -4 \text{ units/sec}$$
$$v = 2 \times 1 = 2 \text{ units/sec}$$

Resultant velocity $$= \sqrt{u^2 + v^2} = \sqrt{(-4)^2 + 2^2}$$

$$= \sqrt{20} = 4.47 \text{ units/sec}$$

and $$\tan\theta = \frac{v}{u} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \qquad \theta = \tan^{-1} .5 = 26° \, 34'$$

$\therefore$ Resultant velocity makes an angle of 26° 34′ with x-axis.

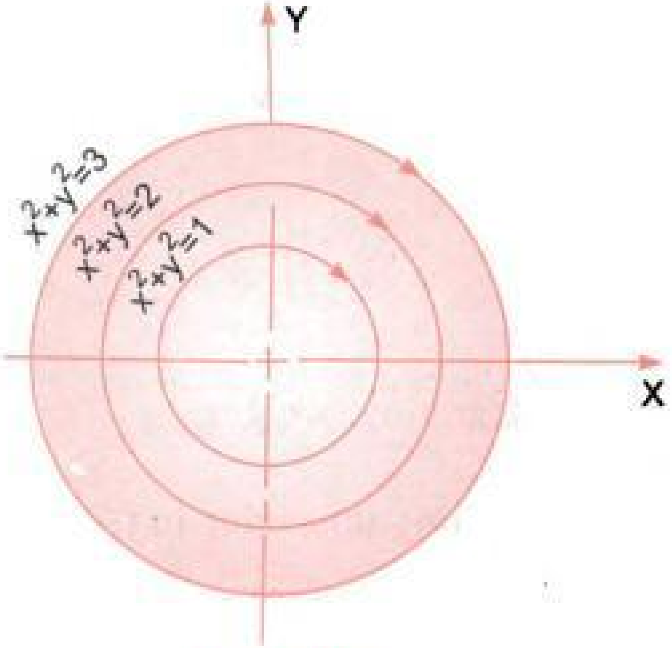

Fig. 5.10

Sketch of Stream Lines.

$$\psi = x^2 + y^2$$

Let $\psi = 1, 2, 3$ and so on.

Then we have
$$1 = x^2 + y^2$$
$$2 = x^2 + y^2$$
$$3 = x^2 + y^2$$

and so on.

Each equation is a equation of a circle. Thus we shall get concentric circles of different diameters as shown in Fig. 5.10.

Problem 5.16 *The velocity components in a two-dimensional flow field for an incompressible fluid are as follows :*

$$u = \frac{y^3}{3} + 2x - x^2 y \text{ and } v = xy^2 - 2y - x^3/3$$

obtain an expression for the stream function ψ. (A.M.I.E., Summer, 1981)

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5.9.3 Angular Deformation or Shear Deformation. It is defined as the average change in the angle contained by two adjacent sides. Let $\Delta\theta_1$ and $\Delta\theta_2$ is the change in angle between two adjacent side of a fluid element as shown in Fig. 5.11 (*c*), then angular deformation or shear strain rate

$$= \frac{1}{2}[\Delta\theta_1 + \Delta\theta_2]$$

Now $$\Delta\theta_1 = \frac{\partial v}{\partial x} \times \frac{\Delta x}{\Delta x} = \frac{\partial v}{\partial x} \text{ and } \Delta\theta_2 = \frac{\partial u}{\partial y} \cdot \frac{\Delta y}{\Delta y} = \frac{\partial u}{\partial y}.$$

$\therefore$ $$\textbf{Angular deformation} = \frac{1}{2}[\Delta\theta_1 + \Delta\theta_2]$$

or $$\textbf{Shear strain rate} = \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] \qquad \text{...(5.16)}$$

5.9.4 Rotation. It is defined as the movement of a fluid element in such a way that both of its axes (horizontal as well as vertical) rotate in the same direction as shown in Fig. 5.11 (*d*). It is equal to $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ for a two-dimensional element in *x*-*y* plane. The rotational components are

$$\left.\begin{aligned} \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \end{aligned}\right\} \qquad \text{...(5.17)}$$

5.9.5 Vorticity. It is defined as the value twice of the rotation and hence it is given as = 2ω.

Problem 5.18 *A fluid flow is given by $V = 8x^3 i - 10x^2 yj$.*

Find the shear strain rate and state whether the flow is rotational or irrotational.

Solution. Given : $V = 8x^3 i - 10x^2 yj$

$\therefore$ $$u = 8x^3, \frac{\partial u}{\partial x} = 24x^2, \frac{\partial u}{\partial y} = 0$$

and $$v = -10x^2 y, \frac{\partial v}{\partial x} = -20xy, \frac{\partial v}{\partial y} = -10x^3$$

(*i*) Shear strain rate is given by equation (5.16) as

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \frac{1}{2}(-20xy + 0) = \mathbf{-10xy.\ Ans.}$$

$$dp = \rho \frac{V^2}{r} dr - \rho g dz \qquad \text{...(5.23)}$$

Equation (5.23) gives the variation of pressure of a rotating fluid in any plane.

5.10.4 Equation of Forced Vortex Flow. For the forced vortex flow, from equation (5.18), we have

$$v = \omega \times r$$

where ω = Angular velocity = Constant.

Substituting the value of v in equation (5.23), we get

$$dp = \rho \times \frac{\omega^2 r^2}{r} dr - \rho g \, dz.$$

Consider two points 1 and 2 in the fluid having forced vortex flow as shown in Fig. 5.14. Integrating the above equation for points 1 and 2, we get

$$\int_1^2 dp = \int_1^2 \rho\omega^2 r dr - \int_1^2 \rho g dz$$

or

$$(p_2 - p_1) = \left[\rho\omega^2 \frac{r^2}{2}\right]_1^2 - \rho g \, [z]_1^2$$

or

$$(p_2 - p_1) = \frac{\rho\omega^2}{2} [r_2^2 - r_1^2] - \rho g \, [z_2 - z_1]$$

$$= \frac{\rho}{2} [\omega^2 r_2^2 - \omega^2 r_1^2] - \rho g \, [z_2 - z_1]$$

$$= \frac{\rho}{2} [v_2^2 - v_1^2] - \rho g \, [z_2 - z_1] \left\{ \begin{array}{l} \because \; v_2 = \omega r_2 \\ \quad\; v_1 = \omega r_1 \end{array} \right\}$$

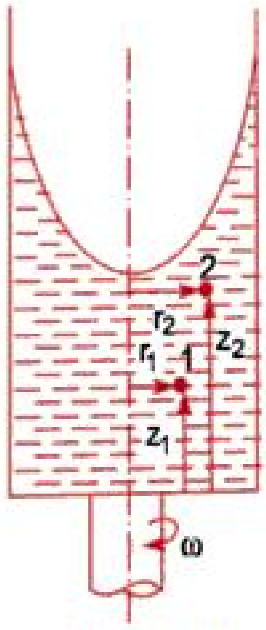

Fig. 5.14

If the points 1 and 2 lie on the free surface of the liquid, then $p_1 = p_2$ and hence above equation becomes

$$0 = \frac{\rho}{2} [v_2^2 - v_1^2] - \rho g \, [z_2 - z_1]$$

or

$$\rho g \, [z_2 - z_1] = \frac{\rho}{2} [v_2^2 - v_1^2]$$

or

$$[z_2 - z_1] = \frac{1}{2g} [v_2^2 - v_1^2].$$

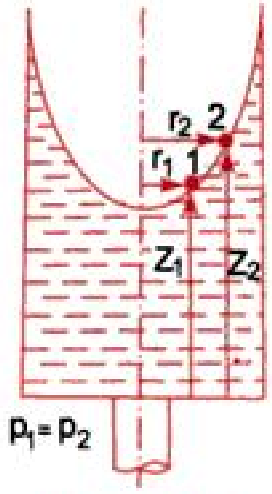

Fig. 5.15

If the point 1 lies on the axis of rotation, then $v_1 = \omega \times r_1 = \omega \times 0 = 0$. The above equation becomes as

$$z_2 - z_1 = \frac{1}{2g} v_2^2 = \frac{v_2^2}{2g}$$

Let $z_2 - z_1 = Z$, then we have $Z = \dfrac{v_2^2}{2g} = \dfrac{\omega^2 \times r_2^2}{2g}$ ...(5.24)

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$\bar{h}$ = C.G. of the wetted area of the sides

$$= \frac{1}{2} \times \text{height of water} = \frac{0.70}{2} = 0.35 \text{ m}$$

∴ Force on the sides before rotation = 1000 × 9.81 × 0.33 × 0.35 = 1133 N

After rotation, the water is upto the top of the cylinder and hence force on the sides

$$= 1000 \times 9.81 \times \text{Wetted area of the sides} \times \frac{1}{2} \times \text{Height of water}$$

$$= 9810 \times \pi D \times 1.0 \times \frac{1}{2} \times 1.0 = 9810 \times \pi \times .15 \times \frac{1}{2} = 2311.43 \text{ N}$$

∴ Difference in pressure on the sides

2311.43 − 1133 = **1178.43 N. Ans.**

5.10.5 Closed Cylindrical Vessels. If a cylindrical vessel is closed at the top, which contains some liquid, the shape of paraboloid formed due to rotation of the vessel will be as shown in Fig. 5.20 for different speed of rotations.

Fig. 5.20 (*a*) shows the initial stage of the cylinder, when it is not rotated. Fig. 5.20 (*b*) shows the shape of the paraboloid formed when the speed of rotation is ω_1. If the speed is increased further say ω_2, the shape of paraboloid formed will be as shown in Fig. 5.20 (*c*). In this case the radius of the parabola at the top of the vessel is unknown. Also the height of the paraboloid formed corresponding to angular speed ω_2 is unknown. Thus to solve the two unknown, we should have two equations. One equation is

$$Z = \frac{\omega_2^2 r^2}{2g}$$

The second equation is obtained from the fact that for closed vessel, volume of air before rotation is equal to the volume of air after rotation.

Volume of air before rotation = Volume of closed vessel − Volume of liquid in vessel

Volume of air after rotation = Volume of paraboloid formed = $\dfrac{\pi r^2 \times Z}{2}$.

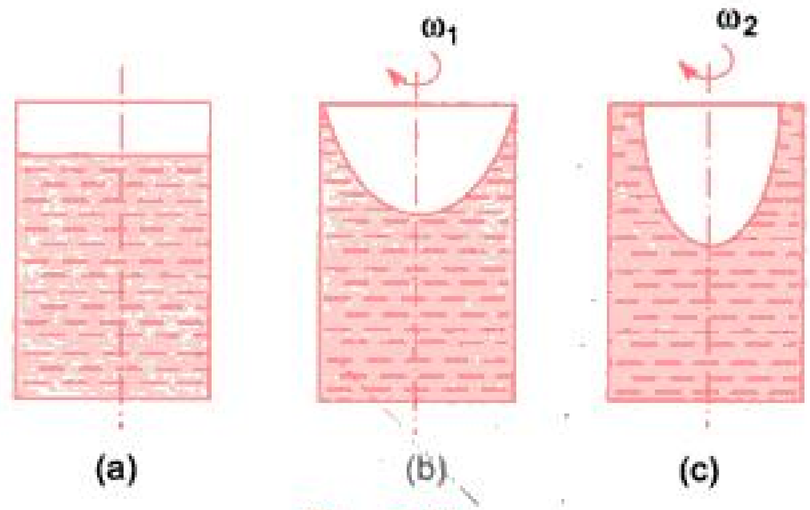

Fig. 5.20

Problem 5.26 *A vessel, cylindrical in shape and closed at the top and bottom, contains water upto a height of 80 cm. The diameter of the vessel is 20 cm and length of vessel 120 cm. The vessel is rotated at a speed of 400 r.p.m. about its vertical axis. Find the height of paraboloid formed.*

$$= 0.573\ [14400 + 7.506\ r_2^4 + 657.6\ r_2^2] - 4.3\ r_2^4$$

$$\frac{12566.3}{0.573} = 21930 = 14400 + 7.506\ r_2^4 + 657.6\ r_2^2 - 4.3\ r_2^4$$

or $$r_2^4\ (7.506 - 4.3) + 657.6\ r_2^2 + 14400 - 21930 = 0$$

or $$3.206\ r_2^4 + 657.6\ r_2^2 - 7530 = 0$$

$\therefore$ $$r_2^2 = \frac{-657.6 \pm \sqrt{657.6^2 - 4 \times (-7530) \times (3.206)}}{2 \times 3.206}$$

$$= \frac{-657.6 \pm \sqrt{432437.76 + 96564.72}}{6.412}$$

$$= \frac{-657.6 \pm 727.32}{6.412} = -\ 215.98 \text{ or } 10.87$$

Negative value is not possible

$\therefore$ $$r_2^2 = 10.87 \text{ cm}^2$$

$\therefore$ Area uncovered at the base $= \pi r_2^2 = \pi \times 10.87 =$ **34.149 cm². Ans.**

Problem 5.29 *A closed cylindrical vessel of diameter 30 cm and height 100 cm contains water upto a depth of 80 cm. The air above the water surface is at a pressure of 5.886 N/cm². The vessel is rotated at a speed of 250 r.p.m. about its vertical axis. Find the pressure head at the bottom of the vessel : (a) at the centre, and (b) at the edge.*

Solution. Given :

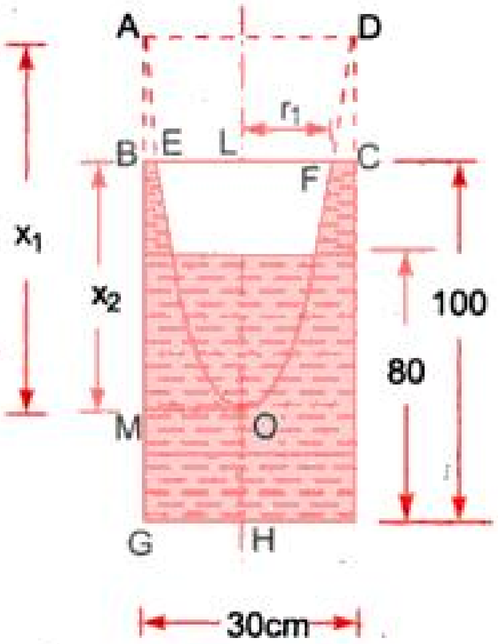

Fig. 5.25

Diameter of vessel $= 30$ cm

$\therefore$ Radius, $R = 15$ cm

Initial height of water, $H = 80$ cm

Length of cylinder, $L = 100$ cm

Pressure of air above water $= 5.886$ N/cm²

or $$p = 5.886 \times 10^4 \ \frac{\text{N}}{\text{m}^2}$$

Head due to pressure, $h = p/\rho g$

$$= \frac{5.886 \times 10^4}{1000 \times 9.81} = 6 \text{ m of water}$$

Speed, $N = 250$ r.p.m.

$\therefore$ $$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.18 \text{ rad/s}$$

Let x_1 = Height of paraboloid formed, if the vessel is assumed open at the top and it is very long.

Then we have $$x_1 = \frac{\omega^2 R^2}{2g} = \frac{26.18^2 \times 15^2}{2 \times 981} = 78.60 \text{ cm} \qquad \ldots(i)$$

Let r_1 is the radius of the actual parabola of height x_2

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Problem 5.32 *In a free cylindrical vortex flow, at a point in the fluid at a radius of 200 mm and at a height of 100 mm, the velocity and pressures are 10 m/s and 117.72 kN/m² absolute. Find the pressure at a radius of 400 mm and at a height of 200 mm. The fluid is air having density equal to 1.24 kg/m³.*

Solution. Point 1 : Given :

Radius, $r_1 = 200 \text{ mm} = 0.20 \text{ m}$

Height, $z_1 = 100 \text{ mm} = 0.10 \text{ m}$

Velocity, $v_1 = 10 \text{ m/s}$

Pressure, $p_1 = 117.72 \text{ kN/m}^2 = 117.72 \times 10^3 \text{ N/m}^2$

At Point 2 : $r_2 = 400 \text{ mm} = 0.4 \text{ m}$

$z_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 =$ pressure at point 2

$\rho = 1.24 \text{ kg/m}^3$

For the free vortex from equation (5.20), we have

$$v \times r = \text{constant or } v_1 r_1 = v_2 r_2$$

$$v_2 = \frac{v_1 \times r_1}{r_2} = \frac{10 \times 0.2}{0.4} = 5 \text{ m/s}$$

Now using equation (5.27), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $\rho = 1.24 \text{ kg/m}^3$

$$\therefore \quad \frac{117.72 \times 10^3}{1.24 \times 9.81} + \frac{10^2}{2 \times 9.81} + 0.1 = \frac{p_2}{\rho g} + \frac{5^2}{2 \times 9.81} + 0.2$$

or
$$\frac{p_2}{\rho g} = \frac{117.72 \times 10^3}{1.24 \times 9.81} + \frac{10^2}{2 \times 9.81} + 0.1 - \frac{5^2}{2 \times 9.81} - 0.2$$

$$= 9677.4 + 5.096 + 0.1 - 1.274 - 0.2 = 9676.22$$

$$p_2 = 9676.22 \times \rho g = 9676.22 \times 1.24 \times 9.81$$

$$= 117705 \text{ N/m}^2 = 117.705 \times 10^3 \text{ N/m}^2$$

$$= 117.705 \text{ kN/m}^2 \text{ (abs.)} = \mathbf{117.705 \text{ kN/m}^2}. \textbf{ Ans.}$$

(B) IDEAL FLOW (POTENTIAL FLOW)

▶ 5.11 INTRODUCTION

Ideal fluid is a fluid which is incompressible and inviscid. Incompressible fluid is a fluid for which density (ρ)remains constant. Inviscid fluid is a fluid for which viscosity (μ) is zero. Hence a fluid for which density is constant and viscosity is zero, is known as an ideal fluid.

The shear stress is given by, $\tau = \mu \dfrac{du}{dy}$. Hence for ideal fluid the shear stress will be zero as $\mu = 0$ for ideal fluid. Also the shear force (which is equal to shear stress multiplied by area) will be zero in

Plotting of Potential lines. For potential lines, the equation is $\phi = U.y + C_2$

Let $\phi = 0$, where $y = 0$. Then $C_2 = 0$.

Hence equation of potential lines becomes as $\phi = U.y$ ...(5.38)

The above equation shows that potential lines are straight lines parallel to x-axis and at a distance of y from the x-axis as shown in Fig. 5.32.

5.14 SOURCE FLOW

The source flow is the flow coming from a point (source) and moving out radially in all directions of a plane at uniform rate. Fig. 5.33 shows a source flow in which the point O is the source from which the fluid moves radially outward. The strength of a source is defined as the volume flow rate per unit depth. The unit of strength of source is m^2/s. It is represented by q.

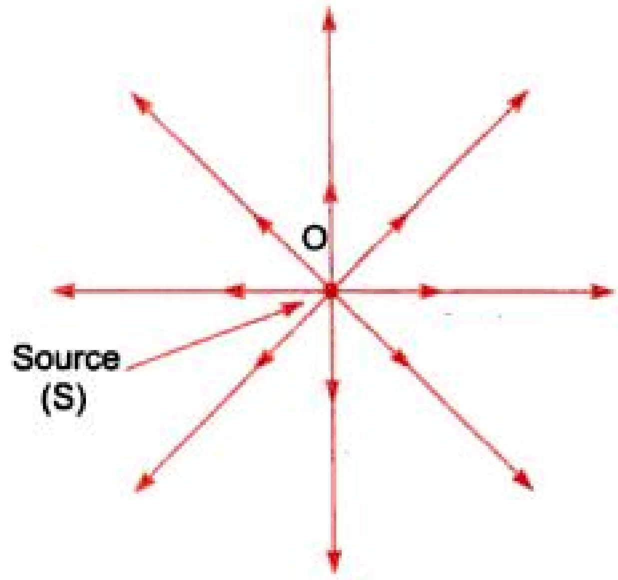

Fig. 5.33 *Source flow (Flow away from source)*

Let u_r = radial velocity of flow at a radius r from the source O

q = volume flow rate per unit depth

r = radius

The radial velocity u_r at any radius r is given by,

$$u_r = \frac{q}{2\pi r} \quad ...(5.39)$$

The above equation shows that with the increase of r, the radial velocity decreases. And at a large distance away from the source, the velocity will be approximately equal to zero. The flow is in radial direction, hence the tangential velocity $u_\theta = 0$.

Let us now find the equation of stream function and velocity potential function for the source flow. As in this case, $u_\theta = 0$, the equation of stream function and velocity potential function will be obtained from u_r.

Equation of Stream Function

By definition, the radial velocity and tangential velocity components in terms of stream function are given by

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \text{ and } u_\theta = -\frac{\partial \psi}{\partial r} \quad \text{[See equation (5.12A)]}$$

But $$u_r = \frac{q}{2\pi r} \quad \text{[See equation (5.39)]}$$

$\therefore$ $$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{q}{2\pi r}$$

or $$d\psi = r.\frac{q}{2\pi r}.d\theta = \frac{q}{2\pi}d\theta$$

Integrating the above equation w.r.t. θ, we get

$$\psi = \frac{q}{2\pi} \times \theta + C_1, \text{ where } C_1 \text{ is a constant of integration.}$$

Let $\psi = 0$, when $\theta = 0$, then $C_1 = 0$.

Hence the equation of stream function becomes as

Problem 5.34 *Determine the velocity of flow at radii of 0.2 m, 0.4 m and 0.8 m, when the water is flowing radially outward in a horizontal plane from a source at a strength of 12 m^2/s.*

Solution. Given :

Strength of source, $q = 12 \text{ m}^2/\text{s}$

The radial velocity u_r at any radius r is given by equation (5.39) as

$$u_r = \frac{q}{2\pi r}$$

When $r = 0.2$ m, $u_r = \frac{12}{2\pi \times 0.2} =$ **9.55 m/s. Ans.**

When $r = 0.4$ m, $u_r = \frac{12}{2\pi \times 0.4} =$ **4.77 m/s. Ans.**

When $r = 0.8$ m, $u_r = \frac{12}{2\pi \times 0.8} =$ **2.38 m/s. Ans.**

Problem 5.35 *Two discs are placed in a horizontal plane, one over the other. The water enters at the centre of the lower disc and flows radially outward from a source of strength 0.628 m^2/s. The pressure, at a radius 50 mm, is 200 kN/m^2. Find :*

(i) pressure in kN/m^2 at a radius of 500 mm and

(ii) stream function at angles of 30° and 60° if $\psi = 0$ at $\theta = 0°$.

Solution. Given :

Source strength, $q = 0.628 \text{ m}^2/\text{s}$

Pressure at radius 50 mm, $p_1 = 200 \text{ kN/m}^2 = 200 \times 10^3 \text{ N/m}^2$

(i) Pressure at a radius 500 mm

Let p_2 = pressure at radius 500 mm

$(u_r)_1$ = velocity at radius 50 mm

$(u_r)_2$ = velocity at radius 500 mm

The radial velocity at any radius r is given by equation (5.39) as

$$u_r = \frac{q}{2\pi r}$$

When $r = 50 \text{ mm} = 0.05$ m, $(u_r)_1 = \frac{0.628}{2\pi \times 0.05} = 1.998 \text{ m/s} \simeq 2 \text{ m/s}$

When $r = 500 \text{ mm} = 0.5$ m, $(u_r)_2 = \frac{0.628}{2\pi \times 0.5} = 0.2 \text{ m/s}$

Applying Bernoulli's equation at radius 0.05 m and at radius 0.5 m,

$$\frac{p_1}{\rho g} + \frac{(u_r)_1^2}{2g} = \frac{p_2}{\rho g} + \frac{(u_r)_2^2}{2g}$$

or
$$\frac{p_1}{\rho} + \frac{(u_r)_1^2}{2} = \frac{p_2}{\rho} + \frac{(u_r)_2^2}{2}$$

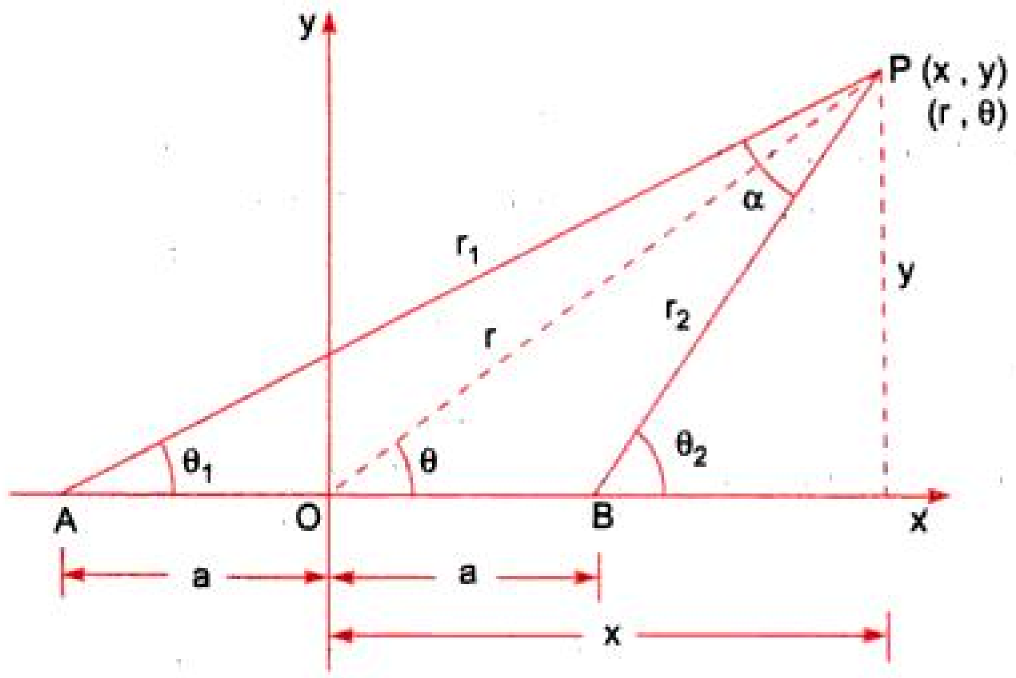

Fig. 5.41

Let r, θ = Cylindrical co-ordinates of point P with respect to origin O

x, y = Corresponding co-ordinates of point P

r_1, θ_1 = Position of point P with respect to source placed at A

r_2, θ_2 = Position of point P with respect to sink placed at B

α = Angle subtended at P by the join of source and sink *i.e.*, angle APB.

Let us find the equation for the resultant stream function and velocity potential function. The equation for stream function due to source is given by equation (5.40) as $\psi_1 = \frac{q \, . \, \theta_1}{2\pi}$ whereas due to sink it is given by $\psi_2 = \frac{(-q\theta_2)}{2\pi}$. The equation for resultant stream function (ψ) will be the sum of these two stream function.

$$\therefore \qquad \psi = \psi_1 + \psi_2$$

$$= \frac{q\theta_1}{2\pi} + \left(\frac{-q\theta_2}{2\pi}\right) = \frac{-q}{2\pi}(\theta_2 - \theta_1)$$

$$= \frac{-q}{2\pi} \, . \, \alpha \quad [\because \quad \alpha = \theta_2 - \theta_1. \text{ In triangle } ABP, \ \theta_1 + \alpha + (180 - \theta_2) = 180^\circ \quad \therefore \quad \alpha = \theta_2 - \theta_1]$$

$$= \frac{-q \, . \, \alpha}{2\pi} \qquad \ldots(5.45)$$

The equation for potential function due to source is given by equation (5.41) as $\phi_1 = \frac{q}{2\pi} \log_e r_1$ and due to sink it is given as $\phi_2 = \frac{-q}{2\pi} \log_e r_2$. The equation for resultant potential function (ϕ) will be the sum of these two potential function.

$$\therefore \qquad \phi = \phi_1 + \phi_2$$

$$= \frac{q}{2\pi} \log_e r_1 + \left(\frac{-q}{2\pi}\right) \log_e r_2$$

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$$= \frac{\pi}{2\pi} \times 0.463 - \frac{8}{2\pi} \times \frac{\pi}{2} \qquad (\because q_1 = 4 \text{ m}^2\text{/s}, q_2 = 8 \text{ m}^2\text{/s})$$

$$= 0.294 - 2.0 = \mathbf{-1.706 \text{ m}^2\text{/s. Ans.}}$$

To find the velocity at the point P, let us first find the stream function in terms of x and y co-ordinates. The stream function in terms of θ_1 and θ_2 is given by equation (*i*) above as

$$\psi = \frac{q_1 \times \theta_1}{2\pi} - \frac{q_2 \times \theta_2}{2\pi}$$

The values of θ_1 and θ_2 in terms of x, y and a are given by equation (5.46*A*) as

$$\tan \theta_1 = \frac{y}{x+a} \quad \text{and} \quad \tan \theta_2 = \frac{y}{(x-a)}$$

or

$$\theta_1 = \tan^{-1} \frac{y}{x+a} \quad \text{and} \quad \theta_2 = \tan^{-1} \frac{y}{(x-a)}$$

Substituting these values of θ_1 and θ_2 in equation (*i*), we get

$$\psi = \frac{q_1}{2\pi} \tan^{-1} \frac{y}{x+a} - \frac{q_2}{2\pi} \tan^{-1} \frac{y}{x-a}$$

The velocity component $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

$$\therefore \qquad u = \frac{\partial \psi}{\partial y}$$

$$= \frac{\partial}{\partial y}\left[\frac{q_1}{2\pi} \tan^{-1} \frac{y}{x+a} - \frac{q_2}{2\pi} \tan^{-1} \frac{y}{x-a}\right]$$

$$= \frac{q_1}{2\pi} \times \frac{1}{1+\left(\frac{y}{x+a}\right)^2} \times \frac{1}{(x+a)} - \frac{q_2}{2\pi} \times \frac{1}{1+\left(\frac{y}{x-a}\right)^2} \times \frac{1}{(x-a)}$$

$$= \frac{q_1}{2\pi} \frac{(x+a)^2}{(x+a)^2 + y^2} \times \frac{1}{(x+a)} - \frac{q_2}{2\pi} \times \frac{(x-a)^2}{(x-a)^2 + y^2} \times \frac{1}{(x-a)}$$

$$= \frac{q_1}{2\pi} \frac{(x+a)}{(x+a)^2 + y^2} - \frac{q_2}{2\pi} \frac{(x-a)}{(x-a)^2 + y^2}$$

At the point $P(1, 1)$, the component u is obtained by substituting $x = 1$ and $y = 1$ in the above equation. The value of a is also equal to one.

$$\therefore \qquad u = \frac{q_1}{2\pi} \frac{1+1}{(1+1)^2 + 1^2} - \frac{q_2}{2\pi} \frac{(1-1)}{(1-1)^2 + 1^2}$$

$$= \frac{q_1}{2\pi} \frac{2}{5} - \frac{q_2}{2\pi} \times 0 = \frac{q_1}{2\pi} \times \frac{2}{5} = \frac{4}{2\pi} \times \frac{2}{5} = 0.2544 \text{ m/s}$$

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$$= \frac{q}{2\pi}\log_e(r+\delta r) - \frac{q}{2\pi}\log_e r = \frac{q}{2\pi}\log_e\left(\frac{r+\delta r}{r}\right) = \frac{q}{2\pi}\log_e\left(1+\frac{\delta r}{r}\right)^*$$

$$= \frac{q}{2\pi}\left[\frac{\delta r}{r} + \left(\frac{\delta r}{r}\right)^2 \times \frac{1}{2} + \ldots\ldots\right]$$

$$= \frac{q}{2\pi}\cdot\frac{\delta r}{r} \qquad \left[\text{As } \frac{\delta r}{r} \text{ is a small quantity. Hence } \left(\frac{\delta r}{r}\right)^2 \text{ becomes negligible}\right]$$

But in Fig. 5.43, from triangle ABC, we get $\frac{\delta r}{2a} = \cos\theta$

$\therefore \qquad \delta r = 2a\cos\theta$

Substituting the value of δr, we get

$$\phi = \frac{q}{2\pi} \times \frac{2a\cos\theta}{r}$$

$$= \frac{\mu}{2\pi} \times \frac{\cos\theta}{r} \qquad [\because 2a \times q = \mu \text{ from equation } (i)] \quad \ldots(5.52)$$

In Fig. 5.43, when $2a \to 0$, the angle $\delta\theta$ becomes very small.

Also $\delta r \to 0$ and AP becomes equal to r. Then

$$\cos\theta = \frac{AD}{AP} = \frac{x}{r}$$

Also $AP^2 = AD^2 + PD^2$ or $r^2 = x^2 + y^2$

Substituting the value of $\cos\theta$ in equation (5.52), we get

$$\phi = \frac{\mu}{2\pi} \times \left(\frac{x}{r}\right) \times \frac{1}{r} = \frac{\mu}{2\pi} \times \frac{x}{r^2}$$

$$= \frac{\mu}{2\pi} \times \frac{x}{(x^2+y^2)} \qquad [\because r^2 = x^2 + y^2]$$

or
$$x^2 + y^2 = \frac{\mu}{2\pi} \times \frac{x}{\phi} \quad \text{or} \quad x^2 + y^2 - \frac{\mu}{2\pi} \times \frac{x}{\phi} = 0$$

The above equation can be written as

$$x^2 - \frac{\mu}{2\pi}\frac{x}{\phi} + \left(\frac{\mu}{4\pi\phi}\right)^2 - \left(\frac{\mu}{4\pi\phi}\right)^2 + y^2 = 0 \qquad \left[\text{Adding and subtracting} \left(\frac{\mu}{4\pi\phi}\right)^2\right]$$

or
$$\left(x - \frac{\mu}{4\pi\phi}\right)^2 + y^2 = \left(\frac{\mu}{4\pi\phi}\right)^2 \qquad \ldots(5.53)$$

The above is the equation of a circle with centre $\left(\frac{\mu}{4\pi\phi}, 0\right)$ and radius $\left(\frac{\mu}{4\pi\phi}\right)$. The centre of the circle lies on x-axis at a distance of $\frac{\mu}{4\pi\phi}$ from y-axis. As the radius of the circle is equal to the distance of the centre of the circle from the y-axis, hence the circle will be tangent to the y-axis.

* Expansion of $\log_e(1+x) = x + \frac{x^2}{2} + \ldots$

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The stream function (ψ) and potential function (ϕ) for the resultant flow are obtained as given below :

ψ = Stream function due to uniform flow + stream function due to source

$$= U \,.\, y + \frac{q}{2\pi}\,\theta \qquad \text{...(5.54)}$$

$$= U \,.\, r \sin\theta + \frac{q}{2\pi}\,\theta \qquad (\because\ y = r\sin\theta)\ \text{...(5.54A)}$$

and ϕ = Velocity potential function due to uniform flow + Velocity potential function due to source

$$= U \,.\, x + \frac{q}{2\pi}\log_e r = U \,.\, r\cos\theta + \frac{q}{2\pi}\log_e r \qquad \text{...(5.54B)}$$

The following are the important points for the resultant flow pattern :

(*i*) *Stagnation point.* On the left side of the source, at the point S lying on the x-axis, the velocity of uniform flow and that due to source are equal and opposite to each other. Hence the net velocity of the combined flow field is zero. This point is known as stagnation point and is denoted by S. The polar co-ordinates of the stagnation point S are r_S and π, where r_S is radial distance of point S from O.

The net velocity (or resultant velocity) is zero at the stagnation point S.

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(U.r\sin\theta + \frac{q}{2\pi}\theta\right) \qquad \left[\because\ \psi = U.r\sin\theta + \frac{q}{2\pi}\theta\right]$$

$$\therefore \qquad = \frac{1}{r}\left[U.r\cos\theta + \frac{q}{2\pi}\right] = U\,.\,\cos\theta + \frac{q}{2\pi r}$$

At the stagnation point, $\theta = \pi$ radians (180°) and $r = r_S$ and net velocity is zero. This means $u_r = 0$ and $v_\theta = 0$. Substituting these values in the above equation, we get

$$0 = U\,.\,\cos 180^\circ + \frac{q}{2\pi r_S} \qquad [\because\ u_r = 0,\ \theta = 180^\circ \text{ and } r = r_S]$$

$$= -U + \frac{q}{2\pi r_S} \quad \text{or} \quad U = \frac{q}{2\pi r_S}$$

or

$$r_S = \frac{q}{2\pi U} \qquad \text{...(5.55)}$$

From the above equation it is clear that position of stagnation point depends upon the free stream velocity U and source strength q. At the stagnation point, the value of stream function is obtained from equation (5.54*A*) as

$$\psi = U\,.\,r\sin\theta + \frac{q}{2\pi}\,.\,\theta$$

For the stagnation point, the above equation becomes as

$$\therefore \qquad \psi_s = U.r_S\sin 180^\circ + \frac{q}{2\pi}\times\theta$$

[$\because$ At stagnation point, $\theta = \pi$ radians = 180° and $r = r_S$]

$$= 0 + \frac{q}{2} = \frac{q}{2} \qquad \text{...(5.56)}$$

The above relation gives the equation of stream line passing through stagnation point. We know that no fluid mass crosses a stream line. Hence a stream line is a *virtual solid surface*.

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$$= 0.75 + 2.5 = \mathbf{3.25\ m^2/s.\ Ans.}$$

(*ii*) *Resultant velocity at P*

The velocity components anywhere in the flow are given by

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\frac{\partial}{\partial\theta}\left[U.r\sin\theta + \frac{q}{2\pi}\theta\right]$$

$$= \frac{1}{r}\left[U.r\cos\theta + \frac{q}{2\pi}\right] = U.\cos\theta + \frac{q}{2\pi r}$$

$$= 3\times\cos 30^\circ + \frac{30}{2\pi\times 0.5} \qquad (\because \text{ At } P,\ r = 0.5,\ \theta = 30^\circ,\ q = 30)$$

$$= 2.598 + 9.55 = 12.14$$

and
$$u_\theta = \frac{-\partial\psi}{\partial r} = -\frac{\partial}{\partial r}\left[U.r\sin\theta + \frac{q}{2\pi}.\theta\right]$$

$$= -U\sin\theta + 0 = -U\sin\theta$$

$$= -3\times\sin 30^\circ = -1.5$$

$\therefore$ Resultant velocity,
$$V = \sqrt{u_r^2 + u_\theta^2}$$

$$= \sqrt{12.14^2 + (-1.5)^2} = \mathbf{12.24\ m/s.\ Ans.}$$

(*iii*) *Location of stagnation point*

The horizontal distance of the stagnation point S from the source is given by equation (5.55) as

$$r_S = \frac{q}{2\pi U} = \frac{30}{2\pi\times 3} = \mathbf{1.59\ m.\ Ans.}$$

The stagnation point will be at a distance of 1.59 m to the left side of the source on the x-axis.

Problem 5.40 *A uniform flow with a velocity of 20 m/s is flowing over a source of strength 10 m²/s. The uniform flow and source flow are in the same plane. Obtain the equation of the dividing stream line and sketch the flow pattern.*

Solution. Given : Uniform velocity, $U = 20$ m/s ; Source strength, $q = 10\ m^2/s$

(*i*) *Equation of the dividing stream line*

The stream function at any point in the combined flow field is given by equation (5.54*A*)

$$\psi = U\,.\,r\sin\theta + \frac{q}{2\pi}\theta$$

$$= 20\times r\sin\theta + \frac{10}{2\pi}\theta \qquad (\because\ U = 20 \text{ m/s and } q = 10\ m^2/s)$$

The value of the stream function for the dividing stream line is $\psi = \frac{q}{2}$. Hence substituting $\psi = \frac{q}{2}$ in the above equation, we get the equation of the dividing stream line.

$\therefore$
$$\frac{q}{2} = 20r\sin\theta + \frac{10}{2\pi}\theta$$

or
$$\frac{10}{2} = 20r\sin\theta + \frac{10}{2\pi}\theta \qquad (\because\ q = 10)$$

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$$= U \times y + \frac{q\theta_1}{2\pi} - \frac{q\theta_2}{2\pi} = U \times y + \frac{q}{2\pi}(\theta_1 - \theta_2)$$

$$= U \times r \sin\theta + \frac{q}{2\pi}(\theta_1 - \theta_2) \qquad (\because \quad y = r\sin\theta) \quad ...(5.59)$$

and ϕ = potential function due to uniform flow + potential function due to source + potential function due to sink

$$= \phi_{\text{uniform flow}} + \phi_{\text{source}} + \phi_{\text{sink}}$$

$$= U \times x + \frac{q}{2\pi}\log_e r_1 + \frac{(-q)}{2\pi}\log_e r_2$$

$$= U \times r\cos\theta + \frac{q}{2\pi}[\log_e r_1 - \log_e r_2] \qquad (\because \quad x = r\cos\theta)$$

$$= U \times r\cos\theta + \frac{q}{2\pi}\left[\log_e \frac{r_1}{r_2}\right] \quad ...(5.60)$$

The followings are the important points for the resultant flow pattern :

(*a*) There will be two stagnation points S_1 and S_2 one to the left of the source and other to the right of the sink. At the stagnation points, the resultant velocity (*i.e.*, velocity due to uniform flow, velocity due to source and velocity due to sink) will be zero. The stagnation point S_1 is to the left of the source and stagnation point S_2 will be to the right of the sink on the x-axis.

Let x_S = Distance of the stagnation points from origin O along x-axis.

Let us calculate this distance x_S.

For the stagnation point S_1,

(*i*) Velocity due to uniform flow = U

(*ii*) Velocity due to source = $\dfrac{q}{2\pi(x_S - a)}$ $\left[\because \text{ The velocity at any radius due to source} = \dfrac{q}{2\pi r};\ \text{For } S_1\text{, the radius from source} = (x_S - a)\right]$

(*iii*) Velocity due to sink = $\dfrac{-q}{2\pi(x_S + a)}$ $[\because$ At S_1, the radius from sink = $(x_S + a)]$

At point S_1, the velocity due to uniform flow is in the positive x-direction whereas due to source and sink are in the –ve x-direction.

$\therefore$ The resultant velocity at $S_1 = U - \dfrac{q}{2\pi(x_S - a)} - \dfrac{(-q)}{2\pi(x_S + a)}$

But the resultant velocity at stagnation point S_1 should be zero.

$$\therefore \qquad U - \frac{q}{2\pi(x_S - a)} + \frac{q}{2\pi(x_S + a)} = 0$$

or

$$U = \frac{q}{2\pi(x_S - a)} - \frac{q}{2\pi(x_S + a)}$$

(*ii*) *Radius of the Rankine circle*

$$R = r = \sqrt{\frac{\mu}{2\pi U}} = \sqrt{\frac{18}{2\pi \times 12}} = \mathbf{0.488\ m.\ Ans.}$$

(*iii*) *Value of stream line function at the Rankine circle*

The value of stream line function (ψ) at the Rankine circle is zero *i.e.*, $\psi = 0$.

(*iv*) *Resultant velocity on the surface of the circle, when $\theta = 30°$*

On the surface of the cylinder, the radial velocity (u_r) is zero. The tangential velocity (u_θ) is given by equation (5.73) as

$$u_\theta = -2U \sin\theta = -2 \times 12 \times \sin 30° = \mathbf{-12\ m/s.\ Ans.}$$

–ve sign shows the clockwise direction of tangential velocity at that point.

$\therefore$ Resultant velocity, $V = \sqrt{u_r^2 + u_\theta^2} = \sqrt{0^2 + (-12)^2} = \mathbf{12\ m/s.\ Ans.}$

(*v*) *Maximum velocity and its location*

The resultant velocity at any point on the surface of the cylinder is equal to u_θ. But u_θ is given by,

$$u_\theta = -2U \sin\theta$$

This velocity will be maximum, when $\theta = 90°$.

$\therefore$ Max. velocity $= -2U = -2 \times 12 = \mathbf{-24\ m/s.\ Ans.}$

Problem 5.44 *A uniform flow of 10 m/s is flowing over a doublet of strength 15 m²/s. The doublet is in the line of the uniform flow. The polar co-ordinates of a point P in the flow field are 0.9 m and 30°. Find : (i) stream line function and (ii) the resultant velocity at the point.*

Solution. Given : $U = 10$ m/s ; $\mu = 15\ m^2/s$; $r = 0.9$ m and $\theta = 30°$.

Let us first find the radius (R) of the Rankine circle. This is given by

$$R = \sqrt{\frac{\mu}{2\pi U}} = \sqrt{\frac{15}{2\pi \times 10}} = 0.488 \text{ m}$$

The polar co-ordinates of the point P are 0.9 m and 30°.

Hence $r = 0.9$ m and $\theta = 30°$.

As the value of r is more than the radius of the Rankine circle, hence point P lies outside the cylinder.

(*i*) *Value of stream line function at the point P*

The stream line function for the composite flow at any point is given by equation (5.69) as

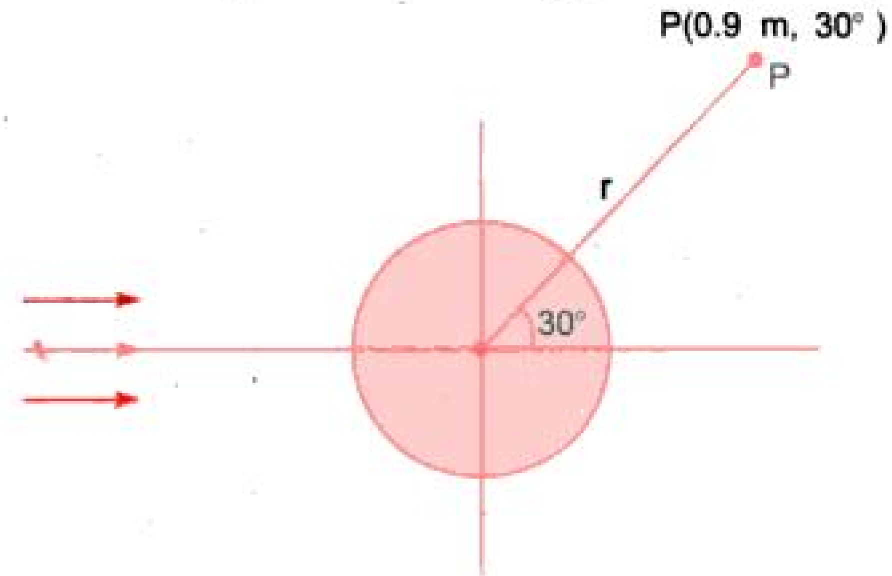

Fig. 5.56

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12. Derive an expression for the depth of paraboloid formed by the surface of a liquid contained in a cylindrical tank which is rotated at a constant angular velocity ω about its vertical axis.
13. Derive an expression for the difference of pressure between two points in a free vortex flow. Does the difference of pressure satisfy Bernoulli's equation ? Can Bernoulli's equation be applied to a forced vortex flow ?
14. Derive, from first principles, the condition for irrotational flow. Prove that, for potential flow, both the stream function and velocity potential function satisfy the Laplace equation. *(A.M.I.E., Summer 1983)*
15. Define velocity potential function and stream function. *(Delhi University, 1987)*
16. Under what conditions can one treat real fluid flow as irrotational (as an approximation). *(A.M.I.E., Winter 1991)*
17. Define the following :
    (*i*) Steady flow, (*ii*) Non-uniform flow,
    (*iii*) Laminar flow, and (*iv*) Two-dimensional flow. *(A.M.I.E., Winter 1989)*
18. (*a*) Distinguish between rotational flow and irrotational flow. Give one example each
    (*b*) Cite two examples of unsteady, non-uniform flow. How can the unsteady flow be transformed to steady flow ? *(J.N.T., University, S 2002)*
19. Explain uniform flow with source and sink. Obtain expressions for stream and velocity potential functions. *(Sastra Deemed University, Nov. 2000)*
20. A point source is a point where an incompressible fluid is imagined to be created and sent out evenly in all directions. Determine its velocity potential and stream function. *(Sastra Deemed University, Nov. 1999, Shanmugha College of Engg.)*
21. (*i*) Explain doublet and define the strength of the doublet
    (*ii*) Distinguish between a source and a sink.
22. Sketch the flow pattern of an ideal fluid flow past a cylinder wih circulation. *(Sastra Deemed University, Nov. 2000)*
23. Show that in case of forced vortex flow, the rise of liquid level at the ends is equal to the fall of liquid level at the axis of rotation. *(R.G.P.V., Bhopal 2000)*
24. Differentiate between :
    (*i*) Stream function and velocity potential function
    (*ii*) Stream line and streak line and
    (*iii*) Rotational and irrotational flows. *(R.G.P.V., Bhopal S 2000)*

(B) NUMERICAL PROBLEMS

1. The diameters of a pipe at the sections 1 and 2 are 15 cm and 20 cm respectively. Find the discharge through the pipe if velocity of water at section 1 is 4 m/s. Determine also the velocity at section 2. [**Ans.** 0.07068 m^3/s, 2.25 m/s]
2. A 40 cm diameter pipe, conveying water, branches into two pipes of diameters 30 cm and 20 cm respectively. If the average velocity in the 40 cm diameter pipe is 3 m/s. Find the discharge in this pipe. Also determine the velocity in 20 cm pipe if the average velocity in 30 cm diameter pipe is 2 m/s. [**Ans.** 0.3769 m^3/s, 7.5 m/s]
3. A 30 cm diameter pipe carries oil of sp. gr. 0.8 at a velocity of 2 m/s. At another section the diameter is 20 cm. Find the velocity at this section and also mass rate of flow of oil. [**Ans.** 4.5 m/s, 113 kg/s]
4. The velocity vector in a fluid flow is given by $V = 2x^3\mathbf{i} - 5x^2y\mathbf{j} + 4t\mathbf{k}$.
   Find the velocity and acceleration of a fluid particle at (1, 2, 3) at time, $t = 1$.
   (Delhi University, June 1996) [**Ans.** 10.95 units, 16.12 units]
5. The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation :

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6

CHAPTER

Dynamics of Fluid Flow

▶ 6.1 INTRODUCTION

In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

▶ 6.2 EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x \qquad \text{...(6.1)}$$

In the fluid flow, the following forces are present :

(*i*) F_g, gravity force.
(*ii*) F_p, the pressure force.
(*iii*) F_v, force due to viscosity.
(*iv*) F_t, force due to turbulence.
(*v*) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

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Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

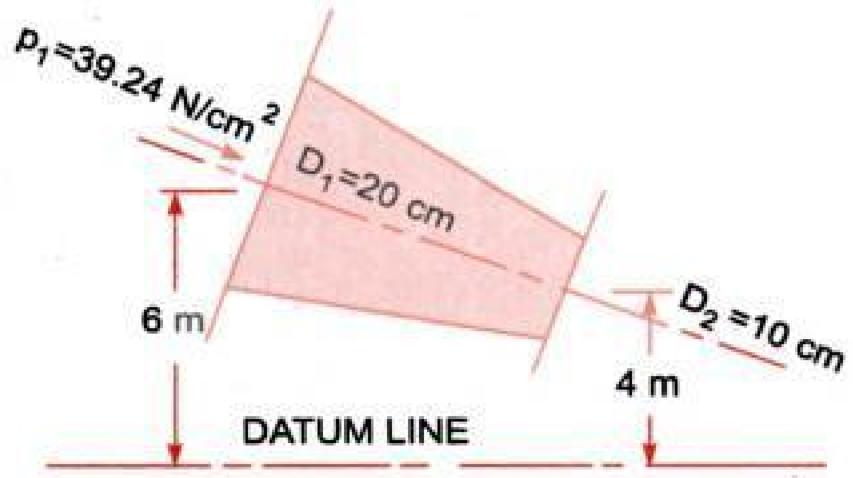

Fig. 6.3

Solution. Given :

At section 1,
$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$
$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$
$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$
$$z_1 = 6.0 \text{ m}$$

At section 2,
$$D_2 = 0.10 \text{ m}$$
$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$
$$z_2 = 4 \text{ m}$$
$$p_2 = ?$$

Rate of flow,
$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$$

Now
$$Q = A_1V_1 = A_2V_2$$

$\therefore$
$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and
$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or
$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or
$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or
$$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$
$$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$
$$p_2 = 41.051 \times 9810 \text{ N/m}^2 = \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2. \text{ Ans.}}$$

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$$= \frac{0.35\,[5-2]^2}{2g} = \frac{0.35 \times 9}{2 \times 9.81} = 0.16 \text{ m}$$

Pressure head, $\frac{p_2}{\rho g} = ?$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2). Then $z_2 = 0$, $z_1 = 2.0$

$$\therefore \quad 2.5 + \frac{5^2}{2 \times 9.81} + 2.0 = \frac{p_2}{\rho g} + \frac{2^2}{2 \times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{p_2}{\rho g} + 0.203 + .16$$

or
$$\frac{p_2}{\rho g} = (2.5 + 1.27 + 2.0) - (.203 + .16)$$

$$= 5.77 - .363 = \mathbf{5.407 \text{ m of fluid. Ans.}}$$

Problem 6.9 *A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 litres/s determine the loss of head and direction of flow.* (A.M.I.E., Summer 1976)

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3\text{/s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \frac{\text{kg}}{\text{m}^3}$

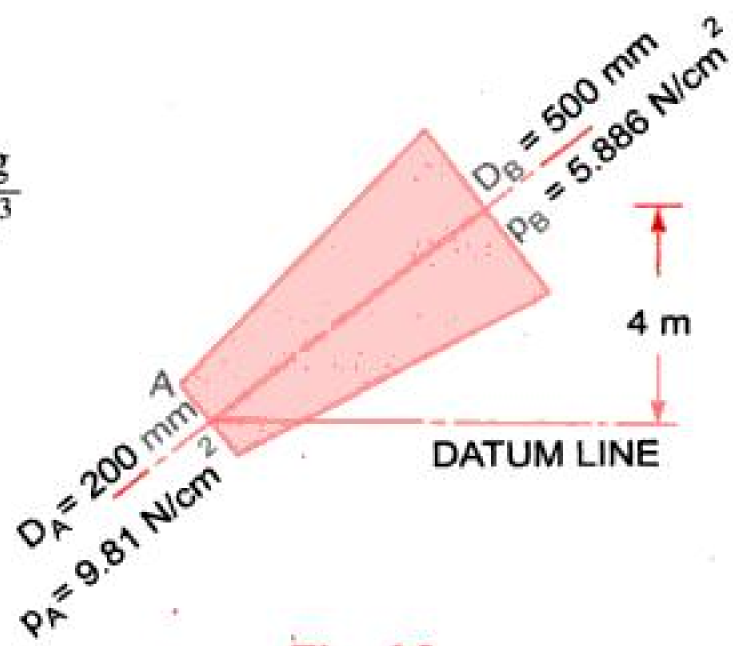

Fig. 6.8

Given : **At section A,** $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \frac{\pi}{4}(D_A)^2 = \frac{\pi}{4}(.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

If datum line is passing through A, then

$Z_A = 0$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \text{ m/s}$$

At section B, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \frac{\pi}{4} D_B{}^2 = \frac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\ [17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $$a_1 = \frac{\pi}{4} d_1^{\,2} = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

$\therefore$ $$a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s. Ans.}}$$

Problem 6.12 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Solution. Given : $d_1 = 20$ cm

$\therefore$ $$a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

(*i*) The discharge, Q of oil $= C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

$$= \mathbf{148.79 \text{ litres/s. Ans.}}$$

(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

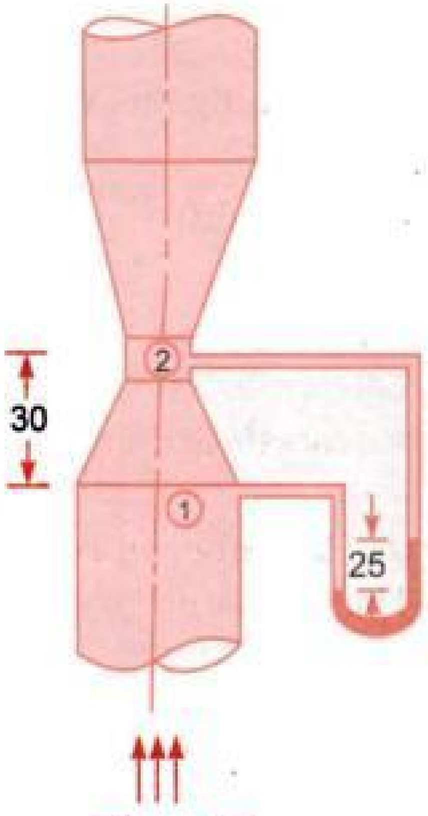

Fig. 6.11

or $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $$z_2 - z_1 = 30 \text{ cm}$$

$\therefore$ $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.77$$

$\therefore$ $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.77 + 30 = 382.77 \text{ cm of oil} = \mathbf{3.8277 \text{ m of oil. Ans.}}$$

or $$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$

$\therefore$ $$(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \ \frac{\text{N}}{\text{m}^2}$$

$$= \frac{33795}{10^4} \text{ N/cm}^2 = \mathbf{3.3795 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 6.20 *Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 litre per second through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm. The co-efficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tappings is 300 mm.*

(i) If two pressure gauges are connected at the tappings such that they are positioned at the levels of their corresponding tapping points, determine the difference of readings in N/cm^2 of the two pressure gauges.

(ii) If a mercury differential manometer is connected, in place of pressure gauges, to the tappings such that the connecting tube upto mercury are filled with oil, determine the difference in the level of the mercury column. (A.M.I.E., Summer 1986)

Solution. Given :

Specific gravity of oil, $S_o = 0.85$

or $$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 = \left[1-\left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2hg$$

$\therefore$ $$v_2 = \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0\, C_c$ $\{\because a_2 = a_0 C_c$ from $(ii)\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}} \qquad \ldots(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ $$C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \qquad \ldots(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Problem 6.22 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.*

$\therefore$ $\overline{V} = 0.80 \times 1.063 = 0.8504$ m/s

Discharge, $Q =$ Area of pipe $\times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem 6.25 *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Solution. Given :

Diff. of mercury level, $x = 100$ mm $= 0.1$ m

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $$h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6 \text{ m of oil}$$

$\therefore$ Velocity of flow $= C_v \sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} = \mathbf{5.49\ m/s.\ Ans.}$

Problem 6.26 *A pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.* (A.M.I.E., Winter, 1979)

Solution. Given :

Stagnation pressure head, $h_s = 6$ m

Static pressure head, $h_t = 5$ m

$\therefore$ $h = 6 - 5 = 1$ m

Velocity of flow, $V = C_v \sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1} = \mathbf{4.34\ m/s.\ Ans.}$

Problem 6.27 *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.* (A.M.I.E., Winter, 1975)

Solution. Given :

Diff. of mercury level, $x = 170$ mm $= 0.17$ m

Sp. gr. of mercury, $S_g = 13.6$

Sp. gr. of sea-water, $S_o = 1.026$

$\therefore$ $$h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$$

$\therefore$ $$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$$

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01\ km/hr.\ Ans.}$$

Problem 6.28 *A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the pitot-*

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F_R is the force exerted on bend. Hence the force required to hold the duct in position is equal to 21746.6 N but it is acting in the opposite direction of F_R. **Ans.**

Problem 6.33 *A pipe of 300 mm diameter conveying 0.30 m³/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm² and 23.544 N/cm².*

Solution. Given :

Dia. of bend, $D = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A = A_1 = A_2 = \frac{\pi}{4}(.3)^2 = 0.07068 \text{ m}^2$

$\therefore$ Discharge, $Q = 0.30 \text{ m}^3/\text{s}$

$\therefore$ Velocity, $V = V_1 = V_2 = \frac{Q}{A} = \frac{0.30}{.07068} = 4.244 \text{ m/s}$

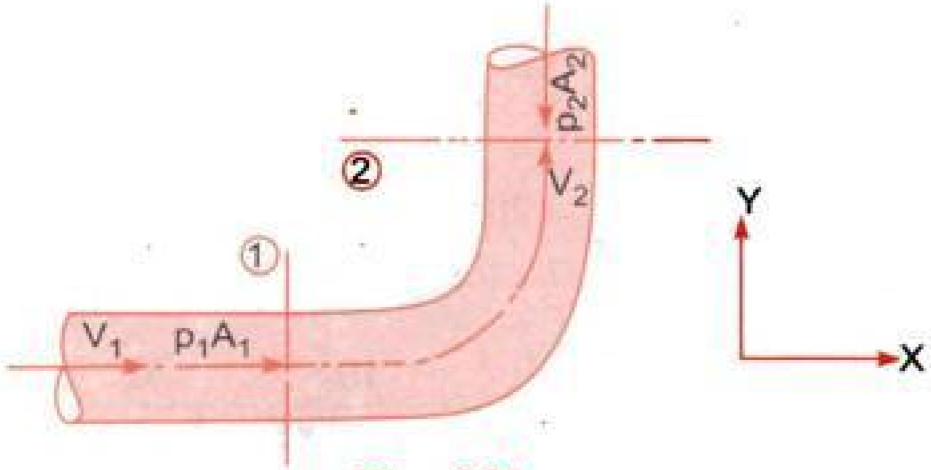

Fig. 6.25

Angle of bend, $\theta = 90°$

$$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2 = 245250 \text{ N/m}^2$$

$$p_2 = 23.544 \text{ N/cm}^2 = 23.544 \times 10^4 \text{ N/m}^2 = 235440 \text{ N/m}^2$$

Force of bend along x-axis $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $\rho = 1000$, $V_{1x} = V_1 = 4.244$ m/s, $V_{2x} = 0$

$(p_1A_1)_x = p_1A_1 = 245250 \times .07068$

$(p_2A_2)_x = 0$

$$\therefore \quad F_x = 1000 \times 0.30\,[4.244 - 0] + 245250 \times .07068 + 0$$
$$= 1273.2 + 17334.3 = 18607.5 \text{ N}$$

Force on bend along y-axis, $F_y = \rho Q\,[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 = 4.244$ m/s

$(p_1A_1)_y = 0$, $(p_2A_2)_y = -p_2A_2 = -235440 \times .07068 = -16640.9$

$$\therefore \quad F_y = 1000 \times 0.30[0 - 4.244] + 0 - 16640.9$$
$$= -1273.2 - 16640.9 = -17914.1 \text{ N}$$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(18607.5)^2 + (17914.1)^2} = -25829.3 \text{ N}$

and $\tan\theta = \frac{F_y}{F_x} = \frac{17914.1}{18607.5} = 0.9627$

$\therefore$ $\theta =$ **43° 54′. Ans.**

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$$-1.0 = 4 - \frac{78.48 \times 2}{U^2} \quad \text{or} \quad \frac{78.48 \times 2}{U^2} = +4.0 + 1.0 = 5.0$$

$\therefore$ $$U^2 = \frac{78.48 \times 2.0}{5.0} = 31.39$$

$\therefore$ $$U = \sqrt{31.39} = 5.60 \text{ m/s}$$

Now the rate of flow of fluid = Area × Velocity of jet

$= A \times U = .001963 \times 5.6 \text{ m}^3\text{/sec}$

$= 0.01099 \approx$ **.011 m³/s. Ans.**

Problem 6.41 *A window, in a vertical wall, is at a distance of 30 m above the ground level. A jet of water, issuing from a nozzle of diameter 50 mm is to strike the window. The rate of flow of water through the nozzle is 3.5 m³/minute and nozzle is situated at a distance of 1 m above ground level. Find the greatest horizontal distance from the wall of the nozzle so that jet of water strikes the window.*

Solution. Given :

Distance of window from ground level = 30 m

Dia. of nozzle, $d = 50$ mm $= 0.05$ m

$\therefore$ Area $$A = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$$

The discharge, $Q = 3.5$ m³/minute

$$= \frac{3.5}{60} = 0.0583 \text{ m}^3\text{/s}$$

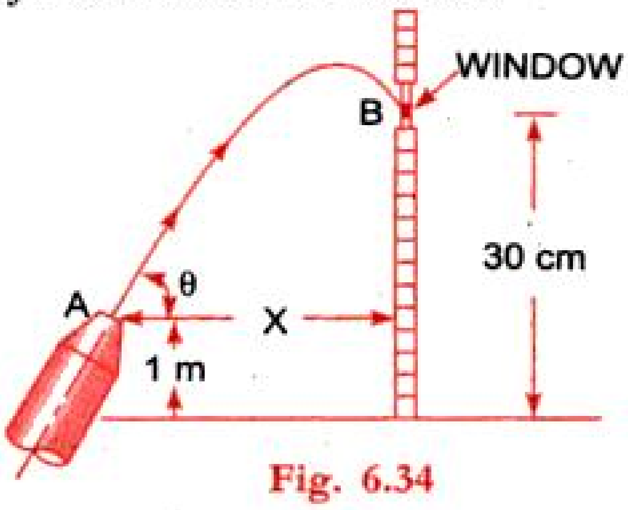

Fig. 6.34

Distance of nozzle from ground = 1 m.

Let the greatest horizontal distance of the nozzle from the wall = x and let angle of inclination = θ. If the jet reaches the window, then the point B on the window is on the centre-line of the jet. The co-ordinates of B with respect to A are

$$x = x,\ y = 30 - 1.0 = 29 \text{ m}$$

The velocity of jet, $$U = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A} = \frac{.0583}{.001963} = 29.69 \text{ m/sec}$$

Using the equation (6.34), which is the equation of jet,

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

or $$29.0 = x \tan\theta - \frac{9.81x^2}{2 - (29.69)^2}\sec^2\theta$$

$$= x\tan\theta - 0.0055 \sec^2\theta \times x^2$$

$$= x\tan\theta - \frac{.0055\,x^2}{\cos^2\theta}$$

$$x\tan\theta - .0055\,x^2/\cos^2\theta - 29 = 0 \qquad \text{...(i)}$$

The maximum value of x with respect to θ is obtained, by differentiating the above equation w.r.t. θ and substituting the value of $\frac{dx}{d\theta} = 0$. Hence differentiating the equation (*i*) w.r.t. θ, we have

$$\left[x\sec^2\theta + \tan\theta \times \frac{dx}{d\theta}\right]$$

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20. What are the different forms of energy in a flowing fluid ? Represent schematically the Bernoulli's equation for flow through a tapering pipe and show the position of total energy line and the datum line. *(Osmania University, 1990)*
21. Write Euler's equation of motion long a streamline and integrate it to obtain Bernoulli's equation. State all assumptions made. *(A.M.I.E., Winter, 1990)*
22. Describe with the help of sketch the construction, operation and use of Pitot-static tube. *(A.M.I.E., Winter, 1988)*
23. Starting with Euler's equation of motion along a stream line, obtain Bernoulli's equation by its integration. List all the assumptions made. *(A.M.I.E., Summer, 1991)*
24. State the different devices that one can use to measure the discharge through a pipe and also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help? *(A.M.I.E., Summer, 1990)*
25. Derive Bernoulli's equation from fundamentals. *(J.N.T.U., Hyderabad, S 2002)*

(B) NUMERICAL PROBLEMS

1. Water is flowing through a pipe of 100 mm diameter under a pressure of 19.62 N/cm^2 (gauge) and with mean velocity of 3.0 m/s. Find the total head of the water at a cross-section, which is 8 m above the datum line. [**Ans.** 28.458 m]
2. A pipe, through which water is flowing is having diameters 40 cm and 20 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 5.0 m/s. Find the velocity head at the sections 1 and 2 and also rate of discharge. [**Ans.** 1.274 m ; 20.387 m ; 0.628 m^3/s]
3. The water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 litres/s. The section 1 is 6 m above datum line and section 2 is 3 m above the datum. If the pressure at section 1 is 29.43 N/cm^2, find the intensity of pressure at section 2. [**Ans.** 32.19 N/cm^2]
4. Water is flowing through a pipe having diameters 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 29.43 N/cm^2 and the pressure at the upper end is 14.715 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 50 lit/s. [**Ans.** 14.618 m]
5. The water is flowing through a taper pipe of length 50 m having diameters 40 cm at the upper end and 20 cm at the lower end, at the rate of 60 litres/s. The pipe has a slope of 1 in 40. Find the pressure at the lower end if the pressure at the higher level is 24.525 N/cm^2. [**Ans.** 25.58 N/cm^2]
6. A pipe of diameter 30 cm carries water at a velocity of 20 m/sec. The pressures at the points *A* and *B* are given as 34.335 N/cm^2 and 29.43 N/cm^2 respectively, while the datum head at *A* and *B* are 25 m and 28 m. Find the loss of head between *A* and *B*. [**Ans.** 2 m]
7. A conical tube of length 3.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 4 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.0 m of liquid. The loss of head in the tube is $0.95\,(v_1 - v_2)^2/2g$, where v_1 is the velocity at the smaller end and v_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in downward direction. [**Ans.** 5.56 m of fluid]
8. A pipe line carrying oil of specific gravity 0.8, changes in diameter from 300 mm at a position *A* to 500 mm diameter to a position *B* which is 5 m at a higher level. If the pressures at *A* and *B* are 19.62 N/cm^2 and 14.91 N/cm^2 respectively, and the discharge is 150 litres/s, determine the loss of head and direction of flow. [**Ans.** 1.45 m, Flow takes place from *A* to *B*]
9. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to inlet and throat is 10 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$. [**Ans.** 88.92 litres/s]

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$$V_A = 0.2/\frac{\pi}{4}(.2^2) = 6.369 \text{ m/s}, \; V_B = 0.2/\frac{\pi}{4}(.5^2) = 1.018 \text{ m/s}$$

$$E_A = (p_A/\rho \times g) + \frac{V_A^2}{2g} + Z_A = (7.848 \times 10^4/1000 \times 9.81) + (6.369^2/2 \times 9.81) + 0 = 10.067 \text{ m}$$

$$E_B = (p_B/\rho \times g) + \frac{V_B^2}{2g} + Z_B = (5.886 \times 10^4/1000 \times 9.81) + (1.018^2/2 \times 9.81) + 3 = 9.052 \text{ m}]$$

41. A venturimeter of inlet diameter 300 mm and throat diameter 150 mm is fixed in a vertical pipe line. A liquid of sp. gr. 0.8 is flowing upward through the pipe line. A differential manometer containing mercury gives a reading of 100 mm when connected at inlet and throat. The vertical difference between inlet and throat is 500 mm. If $C_d = 0.98$, then find : (*i*) rate of flow of liquid in litre per second and (*ii*) difference of pressure between inlet and throat in N/m^2. *(Delhi University, 1988)*

[**Ans.** (*i*) 100 litre/s, (*ii*) 15980 N/m^2]

42. A venturimeter with a throat diameter of 7.5 cm is installed in a 15 cm diameter pipe. The pressure at the entrance to the meter is 70 kPa (gauge) and it is desired that the pressure at any point should not fall below 2.5 m of water absolute. Determine the maximum flow rate of water through the meter. Take $C_d = 0.97$ and atmospheric pressure as 100 kPa. *(J.N.T.U., Hyderabad S 2002)*

[**Hint.** The pressure at the throat will be minimum. Hence $\frac{p_2}{\rho g} = 2.5$ m (abs.)

Given : $d_1 = 15 \text{ cm} \therefore A_1 = \frac{\pi}{4}(15^2) = 176.7 \text{ cm}^2$

$d_2 = 7.5 \text{ cm} \therefore A_2 = \frac{\pi}{4}(7.5^2) = 44.175 \text{ cm}^2$

$p_1 = 70 \text{ kPa} = 70 \times 10^3 \text{ N/m}^2$ (gauge), $p_{atm} = 100 \text{ kPa} = 100 \times 10^3 \text{ N/m}^2$

$\therefore \quad p_1 \text{ (abs.)} = 70 \times 10^3 + 100 \times 10^3 = 170 \times 10^3 \text{ N/m}^2$ (abs.)

$$\therefore \quad \frac{p_1}{\rho g} = \frac{170 \times 10^3}{1000 \times 9.81} = 17.33 \text{ m of water (abs.)}$$

$$\therefore \quad h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 17.33 - 2.5 = 14.83 \text{ m of water} = 1483 \text{ cm of water}$$

Now
$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = \frac{0.97 \times 176.7 \times 44.175 \times \sqrt{2 \times 981 \times 1483}}{\sqrt{176.7^2 - 44 \cdot 175^2}} = 75488 \text{ cm}^3\text{/s}$$

= **75.488 litre/s. Ans.**]

43. Find the discharge of water flowing through a pipe 20 cm diameter placed in an inclined position, where a venturimeter is inserted, having a throat diameter of 10 cm. The difference of pressure between the main and throat is measured by a liquid of specific gravity 0.4 in an inverted *U*-tube, which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of pipe. *(Delhi University, Dec. 2002)*

[**Hint.** Given : $d_1 = 20 \text{ cm} \therefore a_1 = \frac{\pi}{4}(20^2) = 100\pi \text{ cm}^2$; $d_2 = 10 \text{ cm} \therefore a_2 = \frac{\pi}{4}(10^2) = 25\pi \text{ cm}^2$.

$$x = 30 \text{ cm}, \; h = x\left(1 - \frac{S_l}{S_o}\right) = 30\left(1 - \frac{0.4}{1.0}\right) = 18 \text{ cm} = 0.18 \text{ m}$$

But h is also $= \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) \therefore \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 18 \text{ cm} = 0.18 \text{ m}$

7.4.2 Co-efficient of Contraction (C_c). It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice and

a_c = area of jet at vena-contracta.

Then
$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$
$$= \frac{a_c}{a} \qquad \text{...(7.3)}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken 0.64.

7.4.3 Co-efficient of Discharge (C_d). It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$
$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{\text{Actual area}}{\text{Theoretical area}}$$
$$\therefore \quad C_d = C_v \times C_c \qquad \text{...(7.4)}$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

Problem 7.1 *The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.*

Solution. Given :

Head, $H = 10$ cm

Dia. of orifice, $d = 40$ mm $= 0.04$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.04)^2 = .001256 \text{ m}^2$

$C_d = 0.6$

$C_v = 0.98$

(*i*) $$\frac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6$$

But Theoretical discharge $= V_{th} \times$ Area of orifice

V_{th} = Theoretical velocity, where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14$ m/s

$\therefore$ Theoretical discharge $= 14 \times .001256 = 0.01758 \frac{\text{m}^2}{\text{s}}$

$\therefore$ Actual discharge $= 0.6 \times$ Theoretical discharge

$= 0.6 \times .01758 =$ **0.01054 m³/s. Ans.**

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Solution. Given :

Discharge, $Q = 98.2$ lit/s $= 0.0982\ \text{m}^3/\text{s}$

Dia. of orifice, $d = 120$ mm $= 0.12$ m

$\therefore$ Area of orifice, $a = \dfrac{\pi}{4}(.12)^2 = 0.01131\ \text{m}^2$

Head, $H = 10$ m

Horizontal distance of a point on the jet from vena-contracta, $x = 4.5$ m and vertical distance, $y = 0.54$ m

Now theoretical velocity, $V_{th} = \sqrt{2g \times H} = \sqrt{2 \times 9.81 \times 10} = 14.0$ m/s

Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice

$= 14.0 \times .01131 = 0.1583\ \text{m}^3/\text{s}$

The value of C_d is given by $C_d = \dfrac{\text{Actual discharge}}{\text{The oretical discharge}} = \dfrac{Q}{Q_{th}} = \dfrac{0.0982}{0.1583} = \mathbf{0.62.\ Ans.}$

The value of C_c is given by equation (7.6),

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = \mathbf{0.968.\ Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.62}{0.968} = \mathbf{0.64.\ Ans.}$$

Problem 7.6 *A 25 mm diameter nozzle discharges 0.76 m^3 of water per minute when the head is 60 m. The diameter of the jet is 22.5 mm. Determine : (i) the values of co-efficients C_c, C_v and C_d and (ii) the loss of head due to fluid resistance.* (A.M.I.E., Summer 1988)

Solution. Given :

Dia. of nozzle, $D = 25$ mm $= 0.025$ m

Actual discharge, $Q_{act} = 0.76\ \text{m}^3/\text{minute} = \dfrac{0.76}{60} = 0.01267\ \text{m}^3/\text{s}$

Head, $H = 60$ m

Dia. of jet, $d = 22.5$ mm $= 0.0225$ m.

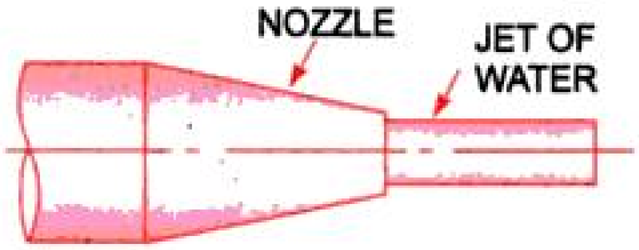

Fig. 7.3

(*i*) *Values of co-efficients :*

Co-efficient of contraction (C_c) is given by,

$$C_c = \frac{\text{Area of jet}}{\text{Area of nozzle}}$$

$$= \frac{\frac{\pi}{4}d^2}{\frac{\pi}{4}D^2} = \frac{d^2}{D^2} = \frac{0.0225^2}{0.025^2} = \mathbf{0.81.\ Ans.}$$

Co-efficient of discharge (C_d) is given by,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

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$$\therefore \quad V_2 = \sqrt{2 \times 9.81 \times 9.5} = 13.652 \text{ m/s}$$

$\therefore$ Rate of flow of water $= C_d \times a_2 \times V_2$

$$= 0.6 \times \frac{\pi}{4} (.1)^2 \times 13.652 \text{ m}^3\text{/s} = \mathbf{0.0643\ m^3/s.\ Ans.}$$

Problem 7.10 *A closed tank partially filled with water upto a height of 0.9 m having an orifice of diameter 15 mm at the bottom of the tank. The air is pumped into the upper part of the tank. Determine the pressure required for a discharge of 1.5 litres/s through the orifice. Take $C_d = 0.62$.*

Solution. Given :

Height of water above orifice, $H = 0.9$ m

Dia. of orifice, $d = 15 \text{ mm} = 0.015$ m

$\therefore$ Area, $a = \frac{\pi}{4}\left[d^2\right] = \frac{\pi}{4} (.015)^2 = 0.0001767 \text{ m}^2$

Discharge, $Q = 1.5 \text{ litres/s} = .0015 \text{ m}^3\text{/s}$

$C_d = 0.62$

Let p is intensity of pressure required above water surface in N/cm^2.

Then pressure head of air $= \frac{p}{\rho g} = \frac{p \times 10^4}{1000 \times 9.81} = \frac{10p}{9.81}$ m of water.

If V_2 is the velocity at outlet of orifice, then

$$V_2 = \sqrt{2g\left(H + \frac{p}{\rho g}\right)} = \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)}$$

$\therefore$ Discharge $Q = C_d \times a \times \sqrt{2g\,(H + p/\rho g)}$

$$.0015 = 0.6 \times .0001767 \times \sqrt{2 \times 9.81\,(0.9 + p/\rho g)}$$

$$\therefore \quad \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)} = \frac{.0015}{0.6 \times .0001767} = 14.148$$

or $$2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right) = 14.148 \times 14.148$$

$$\therefore \quad \frac{10p}{9.81} = \frac{14.148 \times 14.148}{2 \times 9.81} - 0.9 = 10.202 - 0.9 = 9.302$$

$$\therefore \quad p = \frac{9.302 \times 9.81}{10} = \mathbf{9.125\ N/cm^2.\ Ans.}$$

▶ 7.6 FLOW THROUGH LARGE ORIFICES

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by $Q = C_d \times a \times \sqrt{2gh}$. But in case of a large orifice, the velocity is not constant over the entire cross-section of the jet and hence Q cannot be calculated by $Q = C_d \times a \times \sqrt{2gh}$.

Problem 7.14 *Find the discharging through a fully sub-merged orifice of width 2 m if the difference of water levels on both sides of the orifice be 50 cm. The height of water from top and bottom of the orifice are 2.5 m and 2.75 m respectively. Take C_d = 0.6.*

Solution. Given :

Width of orifice, $b = 2$ m

Difference of water level, $H = 50$ cm $= 0.5$ m

Height of water from top of orifice, $H_1 = 2.5$ m

Height of water from bottom of orifice, $H_2 = 2.5$ m

$C_d = 0.6$

Discharge through fully sub-merged orifice is given by equation (7.9)

or
$$Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$$
$$= 0.6 \times 2.0 \times (2.75 - 2.5) \times \sqrt{2 \times 9.81 \times 0.5} \text{ m}^3\text{/s}$$
$$= \mathbf{0.9396\ m^3/s.\ Ans.}$$

Problem 7.15 *Find the discharge through a totally drowned orifice 2.0 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 3 m. Take C_d = 0.62.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1$ m.

Difference of water level on both the sides

$H = 3$ m

$C_d = 0.62$

Discharge through orifice is $Q = C_d \times \text{Area} \times \sqrt{2gH}$
$$= 0.62 \times b \times d \times \sqrt{2gH}$$
$$= 0.62 \times 2.0 \times 1.0 \times \sqrt{2 \times 9.81 \times 3} \text{ m}^3\text{/s} = \mathbf{9.513\ m^3/s.\ Ans.}$$

▶ 7.8 DISCHARGE THROUGH PARTIALLY SUB-MERGED ORIFICE

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. 7.9. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.

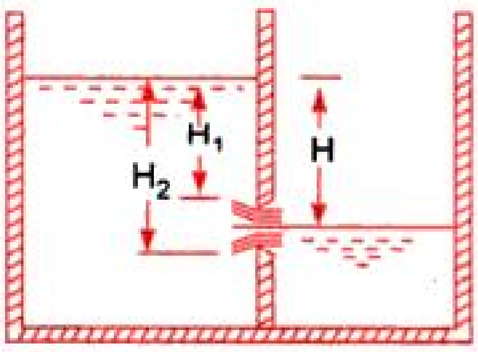

Fig. 7.9 *Partially sub-merged orifice.*

Discharge through the sub-merged portion is given by equation (7.9)
$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

Discharge through the free portion is given by equation (7.8) as

7.10 TIME OF EMPTYING A HEMISPHERICAL TANK

Consider a hemispherical tank of Radius R fitted with an orifice of area 'a' at its bottom as shown in Fig. 7.10. The tank contains some liquid whose initial height is H_1 and in time T, the height of liquid falls to H_2. It is required to find the time T.

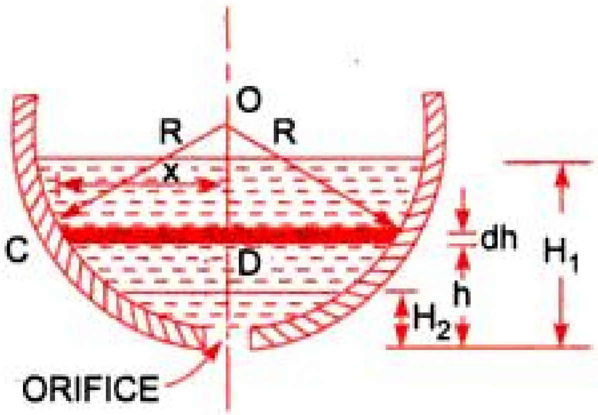

Fig. 7.10 *Hemispherical tank.*

Let at any instant of time, the head of liquid over the orifice is h and at this instant let x be the radius of the liquid surface. Then

Area of liquid surface, $A = \pi x^2$

and theoretical velocity of liquid $= \sqrt{2gh}$.

Let the liquid level falls down by an amount of dh in time dT.

$\therefore$ Volume of liquid leaving tank in time $dT = A \times dh$

$$= \pi x^2 \times dh \quad ...(i)$$

Also volume of liquid flowing through orifice

$$= C_d \times \text{area of orifice} \times \text{velocity} = C_d.a.\sqrt{2gh} \text{ second}$$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d.a.\sqrt{2gh} \times dT \quad ...(ii)$$

From equations (i) and (ii), we get

$$\pi x^2(-dh) = C_d.a.\sqrt{2gh}\ .\ dT$$

– ve sign is introduced, because with the increase of T, h will decrease

$$\therefore \quad -\pi x^2\, dh = C_d.a.\sqrt{2gh}\ .\ dT \quad ...(iii)$$

But from Fig. 7.10, for ΔOCD, we have $OC = R$

$$DO = R - h$$

$$\therefore \quad CD = x = \sqrt{OC^2 - OD^2} = \sqrt{R^2 - (R-h)^2}$$

$$\therefore \quad x^2 = R^2 - (R-h)^2 = R^2 - (R^2 + h^2 - 2Rh) = 2Rh - h^2$$

Substituting x^2 in equation (iii), we get

$$-\pi(2Rh - h^2)dh = C_d.a.\sqrt{2gh}\ .\ dT$$

or

$$dT = \frac{-\pi\left(2Rh - h^2\right)dh}{C_d\,.\,a\,.\sqrt{2gh}} = \frac{-\pi}{C_d\,.\,a\,.\sqrt{2g}}\left(2Rh - h^2\right)h^{-1/2}\ dh$$

$$= \frac{-\pi}{C_d\,.\,a\,.\sqrt{2g}}(2Rh^{1/2} - h^{3/2})dh$$

The total time T required to bring the liquid level from H_1 to H_2 is obtained by integrating the above equation between the limits H_1 to H_2.

$$\therefore \quad T = \int_{H_2}^{H_1} \frac{-\pi}{C_d\,.\,a\,.\sqrt{2g}}(2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d\,.\,a\,.\sqrt{2g}}\int_{H_1}^{H_2}(2Rh^{1/2} - h^{3/2})dh$$

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$\therefore$ Surface area, $A = L \times 2\sqrt{2Rh - h^2}$

$\therefore$ Volume of liquid leaving tank in time dT

$$= A \times dh = 2L\sqrt{2Rh - h^2} \times dh \qquad ...(i)$$

Also the volume of liquid flowing through orifice in time dT

$$= C_d \times \text{Area of orifice} \times \text{Velocity} \times dT$$

But the velocity of liquid at the time considered $= \sqrt{2gh}$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d \times a \times \sqrt{2gh} \times dT \qquad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$2L\sqrt{2Rh - h^2} \times (-dh) = C_d \times a \times \sqrt{2gh \times dT}$$

– ve sign is introduced as with the increase of T, the height h decreases,

$$\therefore \qquad dT = \frac{-2L\sqrt{2Rh - h^2\, dh}}{C_d \times a \times \sqrt{2gh}} = \frac{-2L\sqrt{(2R-h)}\, dh}{C_d \times a \times \sqrt{2g}}$$

[Taking $\sqrt{h}$ common]

$$\therefore \qquad \text{Total time, } T = \int_{H_1}^{H_2} \frac{-2L(2R-h)^{1/2}\, dh}{C_d \times a \times \sqrt{g}}$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \int_{H_1}^{H_2} [2R-h]^{1/2}\, dh\ [2R-h]^{1/2}\, dh$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \left[\frac{(2R-h)^{1/2+1}}{\frac{1}{2}+1} \times (-1) \right]_{H_1}^{H_2}$$

$$= \frac{2L}{C_d \times a \times \sqrt{2g}} \times \frac{2}{3} \times \left[(2R-h)^{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R-H_2)^{3/2} - (2R-H_1)^{3/2} \right] \qquad ...(7.15)$$

For completely emptying the tank, H_2O and hence

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R-H_1)^{3/2} \right]. \qquad ...(7.16)$$

Problem 7.22 *An orifice of diameter 100 mm is fitted at the bottom of a boiler drum of length 5 m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler, given the value of $C_d = 0.6$.*

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Solution. Given :

Dia. of mouthpiece, $d = 150 \text{ mm} = 0.15 \text{ cm}$

$\therefore$ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Head, $H = 6.0 \text{ m}$

$C_d = 0.855$

C_c at vena-contracta = 0.62

Atmospheric pressure head, $H_a = 10.3$ m

$\therefore$ Discharge $= C_d \times a \times \sqrt{2gH}$

$= 0.855 \times .01767 \times \sqrt{2 \times 9.81 \times 6.0} =$ **0.1639 m³/s. Ans.**

Pressure head at vena-contracta

Applying Bernoulli's equation at A and C-C, we get

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

But $\frac{p_A}{\rho g} = H_a + H, v_A = 0,$

$z_A = z_c$

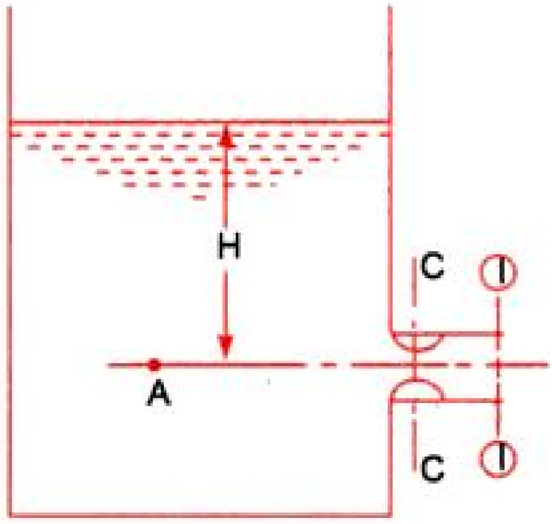

Fig. 7.14

$$\therefore \quad H_a + H + O = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} = H_c + \frac{v_c^2}{2g}$$

$$\therefore \quad H_c = H_a + H - \frac{v_c^2}{2g}$$

But $v_c = \frac{v_1}{0.62}$

$$\therefore \quad H_c = H_a + H\left(\frac{v_1}{.62}\right)^2 \times \frac{1}{2g} = H_a + H - \frac{v_1^2}{2g} \times \frac{1}{(.62)^2}$$

But $\text{H} = 1.375 \; \frac{v_1^2}{2g}$

$$\therefore \quad \frac{v_1^2}{2g} = \frac{H}{1.375} = 0.7272\, H$$

$$\therefore \quad H_c = H_a + H - .7272\, H \times \frac{1}{(.62)^2}$$

$= H_a + H - 1.89\, H = H_a - .89\, H$

$= 10.3 - .89 \times 6.0$ $\{\because H_a = 10.3 \text{ and } H = 6.0\}$

$= 10.3 - 5.34 =$ **4.96 m (Absolute). Ans.**

$$H_a + 0 + H = H_a + \frac{v_1^2}{2g} + 0 + 0.1 \times \frac{v_1^2}{2g} \qquad \left\{\because \frac{p_1}{w} = H_a\right\}$$

$$\therefore \quad H = \frac{v_1^2}{2g} + .1 \times \frac{v_1^2}{2g} = 1.1\,\frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.1}} = \sqrt{\frac{2 \times 9.81 \times 1.5}{1.1}} = 5.1724$$

Now $$Q = A_1 v_1 \text{ or } .005 = \frac{\pi}{4} d_1^2 \times v_1$$

$$\therefore \quad d_1 = \sqrt{\frac{4 \times .005}{\pi \times v_1}} = \sqrt{\frac{4 \times .005}{\pi \times 5.1724}} = 0.035 \text{ m} = \mathbf{3.5 \text{ cm. Ans.}}$$

▶ 7.15 FLOW THROUGH INTERNAL OR RE-ENTRANT ON BORDA'S MOUTHPIECE

A short cylindrical tube attached to an orifice in such a way that the tube projects inwardly to a tank, is called an internal mouthpiece. It is also called Re-entrant or Borda's mouthpiece. If the length of the tube is equal to its diameter, the jet of liquid comes out from mouthpiece without touching the sides of the tube as shown in Fig. 7.16. The mouthpiece is known as *running free*. But if the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to the diameter of mouthpiece at outlet as shown in Fig. 7.17. The mouthpiece is said to be *running full*.

(*i*) **Borda's Mouthpiece Running Free.** Fig. 7.16 shows the Borda's mouthpiece running free.

Let H = height of liquid above the mouthpiece,

a = area of mouthpiece,

a_c = area of contracted jet in the mouthpiece,

v_c = velocity through mouthpiece.

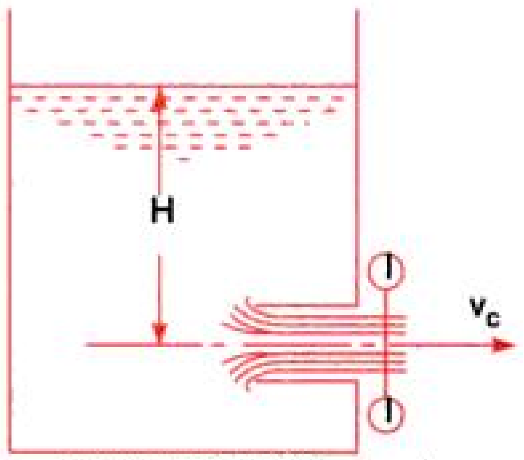

Fig. 7.16

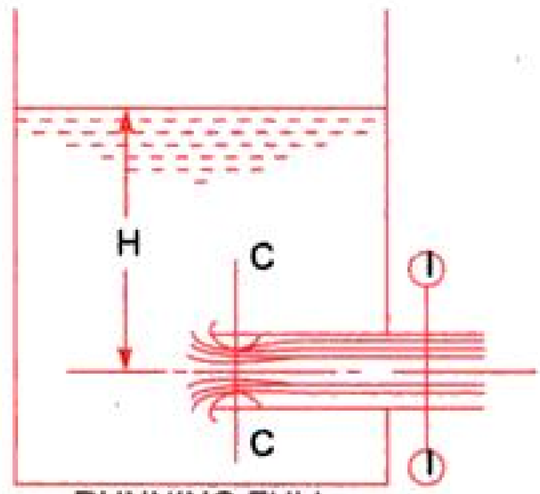

Fig. 7.17

The flow of fluid through mouthpiece is taking place due to the pressure force exerted by the fluid on the entrance section of the mouthpiece. As the area of the mouthpiece is 'a' hence total pressure force on entrance

$$= \rho g \,.\, a \,.\, h$$

where h = distance of C.G. of area 'a' from free surface = H.

$$= \rho g \,.\, a \,.\, H \qquad \ldots(i)$$

According to Newton's second law of motion, the net force is equal to the rate of change of momentum.

a = Area of orifice,

C_d = Co-efficient of discharge.

If the tank is to be completely emptied, then time T,

$$T = \frac{2A\sqrt{H}}{C_d \cdot a \cdot \sqrt{2g}}.$$

9. Time of emptying a hemispherical tank by an orifice fitted at its bottom,

$$T = \frac{\pi}{C_d \cdot a \cdot \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{6}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

and for completely emptying the tank, $T = \dfrac{\pi}{C_d \cdot a \cdot \sqrt{2g}}\left[\dfrac{4}{3}RH_1^{3/2} - \dfrac{5}{2}H_1^{5/2}\right]$

where R = Radius of the hemispherical tank,

H_1 = Initial height of liquid,

H_2 = Final height of liquid,

a = Area of orifice, and

C_d = Co-efficient of discharge.

10. Time of emptying a circular horizontal tank by an orifice at the bottom of the tank,

$$T = \frac{4L}{3C_d \cdot a \cdot \sqrt{2g}}\left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}\right]$$

and for completely emptying the tank, $T = \dfrac{4L}{3C_d \cdot a \cdot \sqrt{2g}}\left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]$

where L = Length of horizontal tank.

11. Co-efficient of discharge for,

(*i*) External mouthpiece, $C_d = 0.855$

(*ii*) Internal mouthpiece, running full, $C_d = 0.707$

(*iii*) Internal mouthpiece running free, $C_d = 0.50$

(*iv*) Convergent or convergent-divergent, $C_d = 1.0$.

12. For an external mouthpiece, absolute pressure head at vena-contracta

$$H_c = H_a - 0.89\ H$$

where H_a = atmospheric pressure head = 10.3 m of water

H = head of liquid above the mouthpiece.

13. For a convergent-divergent mouthpiece, the ratio of area's at outlet and at vena-contracta is

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}}$$

where a_1 = Area of mouthpiece at outlet

a_c = Area of mouthpiece at vena-contracta

H_a = Atmospheric pressure head

H_c = Absolute pressure head at vena-contracta

H = Height of liquid above mouthpiece.

14. In case of internal mouthpieces, if the jet of liquid comes out from mouthpiece without touching its sides it is known as running free. But if the jet touches the sides of the mouthpiece, it is known as running full.

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8

CHAPTER

Notches and Weirs

▶ 8.1 INTRODUCTION

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

▶ 8.2 CLASSIFICATION OF NOTCHES AND WEIRS

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :
   (*i*) Rectangular weir,
   (*ii*) Triangular weir, and
   (*iii*) Trapezoidal weir (Cippoletti weir)

(*b*) According to the shape of the crest :
   (*i*) Sharp-crested weir,
   (*ii*) Broad-crested weir,
   (*iii*) Narrow-crested weir, and
   (*iv*) Ogee-shaped weir.

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H.H^{3/2} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right]$$

$$= \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \qquad \text{...(8.2)}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan \frac{\theta}{2} = 1$$

Discharge $$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \qquad \text{...(8.3)}$$

$$= 1.417\ H^{5/2}.$$

Problem 8.4 *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a V-notch is given by equation (8.2)

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan \frac{60}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.8182 \times 0.0493 = \mathbf{0.040\ m^3/s.\ Ans.}$$

Problem 8.5 *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

(Osmania University, 1990 ; A.M.I.E., Winter, 1975)

Solution. Given :

For rectangular weir, length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation (8.1) as

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Consider a stepped notch as shown in Fig. 8.6.

Let H_1 = Height of water above the crest of notch (1),

L_1 = Length of notch 1,

H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

C_d = Co-efficient of discharge for all notches

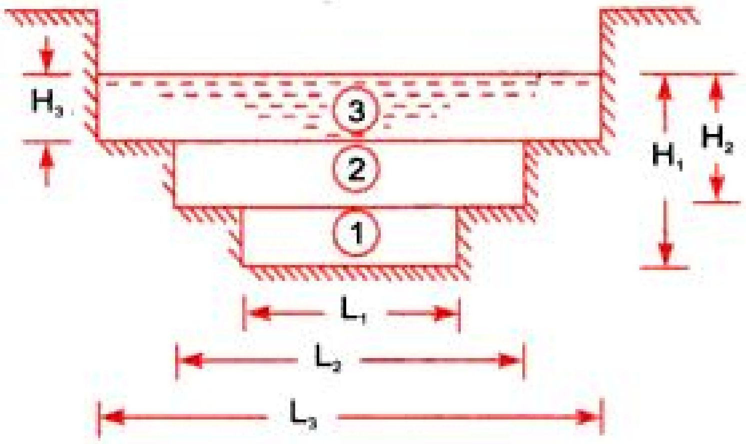

Fig. 8.6 *The stepped notch.*

$\therefore$ Total discharge $Q = Q_1 + Q_2 + Q_3$

or
$$Q = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$+ \frac{2}{3} C_d \times L_2 \times \sqrt{2g}\,[H_2^{3/2} - H_3^{3/2}] + \frac{2}{3} C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}. \quad ...(8.5)$$

Problem 8.8 *Fig. 8.7 shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62.*

Solution. Given :

$L_1 = 40$ cm, $L_2 = 80$ cm,

$L_3 = 120$ cm

$H_1 = 50 + 30 + 15 = 95$ cm,

$H_2 = 80$ cm, $H_3 = 50$ cm,

$C_d = 0.62$

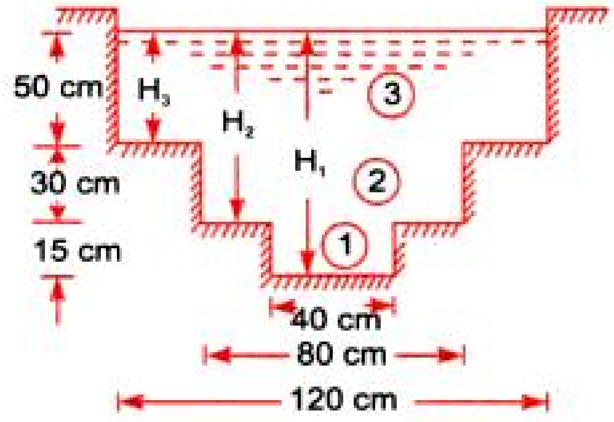

Fig. 8.7

Total discharge, $Q = Q_1 + Q_2 + Q_3$

where
$$Q_1 = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 981} \times [95^{3/2} - 80^{3/2}]$$

$$= 732.26[925.94 - 715.54] = 154067 \text{ cm}^3\text{/s} = 154.067 \text{ lit/s}$$

$$Q_2 = \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \times [H_2^{3/2} - H_3^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 981} \times [80^{3/2} - 50^{3/2}]$$

$$= 1464.52[715.54 - 353.55] \text{ cm}^3\text{/s} = 530141 \text{ cm}^3\text{/s} = 530.144 \text{ lit/s}$$

and
$$Q_3 = \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 981} \times 50^{3/2} = 776771 \text{ cm}^3\text{/s} = 776.771 \text{ lit/s}$$

$$\therefore \quad Q = Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771$$

= **1460.98 lit/s. Ans.**

But $$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}$$

$\therefore$ $$-Adh = \frac{2}{3} C_d \times L \times \sqrt{2g} \,.\, h^{3/2} \times dT \text{ or } dT = \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

The total time T is obtained by integrating the above equation between the limits H_1 to H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

or $$T = \frac{-A}{\frac{2}{3} C_d \times L \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-3/2}\, dh = \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-3/2+1}}{-\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-1/2}}{-\frac{1}{2}} \right]_{H_1}^{H_2} = \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left(-\frac{2}{1} \right) \left[\frac{1}{\sqrt{h}} \right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]. \quad \text{...(8.8)}$$

(b) TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A TRIANGULAR WEIR OR NOTCH

Consider a reservoir or tank of uniform cross-sectional area A, having a triangular weir or notch in one of its sides.

Let θ = Angle of the notch

C_d = Co-efficient of discharge

H_1 = Initial height of liquid above the apex of notch

H_2 = Final height of liquid above the apex of notch

T = Time required in seconds, to lower the height from H_1 to H_2 above the apex of the notch.

Let at any instant, the height of liquid surface above the apex of weir or notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then

$$-Adh = Q \times dT$$

–ve sign is taken, as with the increase of T, h decreases.

And Q for a triangular notch is

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \sqrt{2g} \times h^{5/2}$$

$\therefore$ $$-Adh = \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2} \times dT$$

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$$= 1.098 \times 0.0894 = 0.0982 \text{ m}^3/\text{s}$$

Velocity of approach, $V_a = \dfrac{Q}{A} = \dfrac{.0982}{0.75} = 0.1309$ m/s

$\therefore$ Additional head, $h_a = \dfrac{V_a^2}{2g} = (.1309)^2/2 \times 9.81 = .0008733$ m

Then discharge with velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81}\ [(0.2 + .00087)^{3/2} - (.00087)^{3/2}]$$

$$= 1.098\ [0.09002 - .00002566]$$

$$= 1.098 \times 0.09017 = \mathbf{.09881\ m^3/s.\ Ans.}$$

Problem 8.16 *Find the discharge over a rectangular weir of length 100 m. The head of water over the weir is 1.5 m. The velocity of approach is given as 0.5 m/s. Take $C_d = 0.60$.*

Solution. Given :

Length of weir, $L = 100$ m

Head of water, $H_1 = 1.5$ m

Velocity of approach, $V_a = 0.5$ m/s

$C_d = 0.60$

$\therefore$ Additional head, $h_a = \dfrac{V_a^{\ 2}}{2g} = \dfrac{0.5 \times 0.5}{2 \times 9.81} = 0.0127$ m

The discharge, Q over a rectangular weir due to velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 100 \times \sqrt{2 \times 9.81}\ [(1.5 + .0127)^{3/2} - .0127^{3/2}]$$

$$= 177.16\ [\ 1.5127^{3/2} - .0127^{3/2}]$$

$$= 177.16\ [1.8605 - .00143] = \mathbf{329.35\ m^3/s.\ Ans.}$$

Problem 8.17 *A rectangular weir of crest length 50 cm is used to measure the rate of flow of water in a rectangular channel of 80 cm wide and 70 cm deep. Determine the discharge in the channel if the water level is 80 mm above the crest of weir. Take velocity of approach into consideration and value of $C_d = 0.62$.*

Solution. Given :

Length of weir, $L = 50$ cm $= 0.5$ m

Area of channel, A = Width × depth = 80 cm × 70 cm = $0.80 \times 0.70 = 0.56$ m^2

Head over weir, $H = 80$ mm $= 0.08$ m

$C_d = 0.62$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1)

Solution. Given :

Head of water, $H = 40 \text{ cm} = 0.40 \text{ m}$

Length of weir, $L = 1.5 \text{ m}$

(*i*) Francis's Formula for end contraction suppressed is given by equation (8.12).

$$Q = 1.84\, L \times H^{3/2} = 1.84 \times 1.5 \times (.40)^{3/2}$$
$$= 0.6982 \text{ m}^3\text{/s}$$

(*ii*) Bazin's Formula is given by equation (8.14)

$$Q = m \times L \times \sqrt{2g} \times H^{3/2}$$

where $m = 0.405 + \dfrac{.003}{H} = 0.405 + \dfrac{.003}{.40} = 0.4125$

$$\therefore \quad Q = .4125 \times 1.5 \times \sqrt{2 \times 9.81} \times (.4)^{3/2}$$
$$= \mathbf{0.6932\ m^3/s.\ Ans.}$$

Problem 8.21 *A weir 36 metres long is divided into 12 equal bays by vertical posts, each 60 cm wide. Determine the discharge over the weir if the head over the crest is 1.20 m and velocity of approach is 2 metres per second.* (A.M.I.E., Summer, 1978)

Solution. Given :

Length of weir, $L_1 = 36 \text{ m}$

Number of bays, $= 12$

For 12 bays, no. of vertical post $= 11$

Width of each post $= 60 \text{ cm} = 0.6 \text{ m}$

$\therefore$ Effective length, $L = L_1 - 11 \times 0.6 = 36 - 6.6 = 29.4 \text{ m}$

Head on weir, $H = 1.20 \text{ m}$

Velocity of approach, $V_a = 2 \text{ m/s}$

$\therefore$ Head due to V_a, $h_a = \dfrac{V_a^2}{2g} = \dfrac{2^2}{2 \times 9.81} = 0.2038 \text{ m}$

Number of end contraction, $n = 2 \times 12$ {Each bay has two end contractions}

$= 24$

$\therefore$ Discharge by Francis Formula with end contraction and velocity of approach is

$$Q = 1.84\,[L - 0.1 \times n(H + h_a)][(H + h_a)^{3/2} - h_a^{3/2}]$$
$$= 1.84[29.4 - 0.1 \times 24(1.20 + .2038)] \times [(1.2 + .2038)^{1.5} - .2038^{1.5}]$$
$$= 1.84[29.4 - 3.369][1.663 - .092]$$
$$= \mathbf{75.246\ m^3/s.\ Ans.}$$

Problem 8.22 *A discharge of 2000 m³/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 10 m. If the velocity of approach is 4 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 2 m.*

Solution. Given :

Total discharge, $Q = 2000 \text{ m}^3\text{/s}$

Length of each opening, $L = 10$

Velocity of approach, $V_a = 4 \text{ m/s}$

The discharge will be maximum, if $(Hh^2 - h^2)$ is maximum

or $$\frac{d}{dh}(Hh^2 - h^3) = 0 \text{ or } 2h \times H - 3h^3 = 0 \text{ or } 2H = 3h$$

$$\therefore \qquad h = \frac{2}{3}H$$

Q_{max} will be obtained by substituting this value of h in equation (8.18) as

$$Q_{max} = C_d \times L \times \sqrt{2g\left[H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3\right]}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{H \times \frac{4}{9} \times H^2 - \frac{8}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d \times L \times \sqrt{2g}\sqrt{\frac{(12-8)H^3}{27}}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{27}H^3} = C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= .3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2} = 1.7047 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times C_d \times L \times H^{3/2}. \qquad \text{...(8.19)}$$

▶ 8.14 DISCHARGE OVER A NARROW-CRESTED WEIR

For a narrow-crested weir, $2L < H$. It is similar to a rectangular weir or notch hence, Q is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.20)}$$

▶ 8.15 DISCHARGE OVER AN OGEE WEIR

Fig. 8.15 shows an Ogee weir, in which the crest of the weir rises up to maximum height of $0.115 \times H$ (where H is the height of water above inlet of the weir) and then falls as shown in Fig. 8.11. The discharge for an Ogee weir is the same as that of a rectangular weir, and it is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.21)}$$

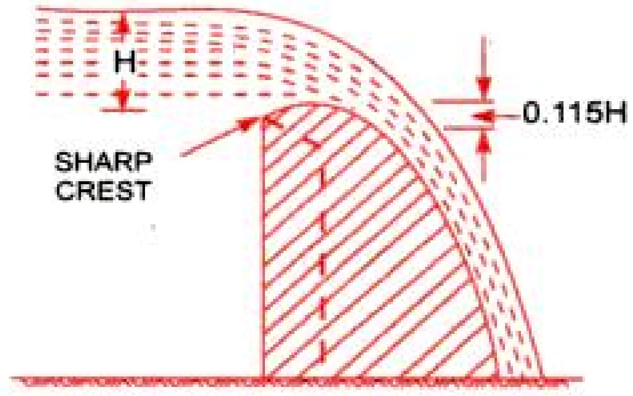

Fig. 8.11 *An Ogee weir.*

▶ 8.16 DISCHARGE OVER SUBMERGED OR DROWNED WEIR

When the water level on the downstream side of a weir is above the crest of the weir, then the weir is called to be a submerged or drowned weir. Fig. 8.12 shows a submerged weir. The total discharge, over the weir is obtained by dividing the weir into two parts. The portion between upstream and downstream water surface may be treated as free weir and portion between downstream water surface and crest of weir as a drowned weir.

$$Q = m\,L\,\sqrt{2g}H^{3/2} \qquad \text{... without velocity of approach}$$

$$= m\,L\,\sqrt{2g}\,[(H + h_a)^{3/2}] \qquad \text{... with velocity of approach}$$

where $m = \frac{2}{3}\,C_d = 0.405 + \frac{.003}{H}$ ... without velocity of approach

$$= 0.405 + \frac{.003}{(H + h_a)} \qquad \text{... with velocity of approach.}$$

12. A trapezoidal weir, with side slope or 1 horizontal to 4 vertical, is called Cipolletti weir. The discharge through Cipolletti weir is given by

$$Q = \frac{2}{3}\,C_d \times L \times \sqrt{2g}\,H^{3/2} \qquad \text{... without velocity of approach}$$

$$= \frac{2}{3}\,C_d \times L \times \sqrt{2g}\,[(H + h_a)^{3/2} - h_a^{3/2}] \qquad \text{... with velocity of approach.}$$

13. The discharge over a Broad-crested weir is given by,

$$Q = C_d L\,\sqrt{2g\left(Hh^2 - h^3\right)}$$

where H = height of water, above the crest
h = head of water at the middle of the weir which is constant
L = length of the weir.

14. The condition for maximum discharge over a Board-crested weir is $h = \frac{2}{3}\,H$

and maximum discharge is given by $Q_{max} = 1.705\,C_d\,L\,H^{3/2}$.

15. The discharge over an Ogee weir is given by $Q = \frac{2}{3}\,C_d L \times \sqrt{2g} \times H^{3/2}$.

16. The discharge over sub-merged or drowned weir is given by

Q = discharge over upper portion + discharge through downed portion

$$= \frac{2}{3}\,C_{d_1}\,Lh \times \sqrt{2g}\,(H - h)^{3/2} + C_{d_2}\,Lh \times \sqrt{2g\,(H - h)}$$

where H = height of water on the upstream side of the weir,
h = height of water on thedownstream side of the weir.

EXERCISE 8

(A) THEORETICAL PROBLEMS

1. Define the terms : notch, weir, nappe and crest.
2. How are the weirs and notches classified ?
3. Find an expression for the discharge over a rectangular weir in terms of head of water over the crest of the weir.
4. Prove that the discharge through a triangular notch or weir is given by

$$Q = \frac{8}{15}\,C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\,H^{3/2}$$

where H = head of water over the notch or weir
θ = angle of notch or weir.

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in Fig. 9.1 (*a*). Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius $(r + dr)$. Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are :

1. The pressure force, $p \times \pi r^2$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r\Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero *i.e.*,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} \cdot r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \qquad ...(9.1)$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown in Fig. 9.2 (*a*).

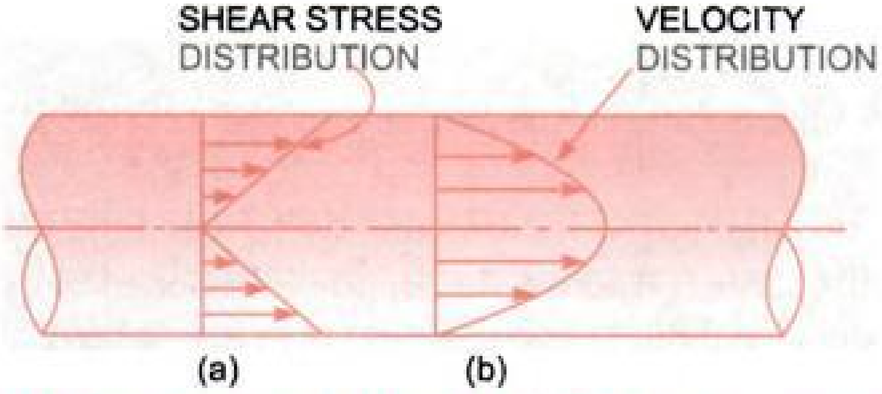

Fig. 9.2 *Shear stress and velocity distribution across a section.*

(*i*) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in equation (9.1).

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$\therefore$
$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting this value in (9.1), we get

Problem 9.2 *An oil of viscosity 0.1 Ns/m² and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.*

(Delhi University, 1987)

Solution. Given : Viscosity, $\mu = 0.1$ Ns/m²

Relative density $= 0.9$

$\therefore$ ρ_0 or density of oil $= 0.9 \times 1000 = 900$ kg/m³ ($\because$ Density of water = 1000 kg/m³)

$$D = 50 \text{ mm} = .05 \text{ m}$$

$$L = 300 \text{ m}$$

$$Q = 3.5 \text{ litres/s} = \frac{3.5}{1000} = .0035 \text{ m}^3/\text{s}$$

Find (*i*) Pressure drop, $p_1 - p_2$

(*ii*) Shear stress at pipe wall, τ_0

(*i*) **Pressure drop $(p_1 - p_2)$** $= \dfrac{32\mu\bar{u}L}{D^2}$, where $\bar{u} = \dfrac{Q}{\text{Area}} = \dfrac{.0035}{\frac{\pi}{4}D^2} = \dfrac{.0035}{\frac{\pi}{4}(.05)^2} = 1.782$ m/s

The Reynolds number (R_e) is given by, $R_e = \dfrac{\rho V D}{\mu}$

where $\rho = 900$ kg/m³, V = average velocity $= \bar{u} = 1.782$ m/s

$$\therefore \qquad R_e = 900 \times \frac{1.782 \times .05}{0.1} = 801.9$$

As Reynold number is less than 2000, the flow is viscous or laminar

$$\therefore \qquad p_1 - p_2 = \frac{32 \times 0.1 \times 1.782 \times 3000}{(.05)^2}$$

$$= 684288 \text{ N/m}^2 = 68428 \times 10^{-4} \text{ N/cm}^2 = \mathbf{68.43\ N/cm^2.\ Ans.}$$

(*ii*) **Shear Stress at the pipe wall (τ_0)**

The shear stress at any radius r is given by the equation (9.1)

i.e.,
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$\therefore$ Shear stress at pipe wall, where $r = R$ is given by

$$\tau_0 = \frac{-\partial p}{\partial x}\frac{R}{2}$$

Now
$$\frac{-\partial p}{\partial x} = \frac{-(p_2 - p_1)}{x_2 - x_1} = \frac{p_1 - p_2}{x_2 - x_1} = \frac{p_1 - p_2}{L}$$

$$= \frac{684288}{300}\frac{\text{N/m}^2}{\text{m}} = 2280.96 \text{ N/m}^3$$

and
$$R = \frac{D}{2} = \frac{.05}{2} = .025 \text{ m}$$

$$\tau_0 = 2280.96 \times \frac{.025}{2}\frac{\text{N}}{\text{m}^2} = \mathbf{28.512\ N/m^2.\ Ans.}$$

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$$\therefore \qquad \frac{\partial p}{\partial x} = -\frac{196.2 \times 4}{0.1} = -7848 \text{ N/m}^2 \text{ per m}$$

$\therefore$ Pressure Gradient = **– 7848 N/m² per m. Ans.**

(*ii*) **Average velocity, $\bar{u}$**

$$\bar{u} = \frac{1}{2} U_{max} = \frac{1}{2}\left[-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2\right] \qquad \left\{\because \; U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x} R^2\right\}$$

$$= \frac{1}{8\mu} - \left(-\frac{\partial p}{\partial x}\right) R^2$$

$$= \frac{1}{8 \times 0.7} \times (7848) \times (.05)^2 \qquad \left\{\because \; R = \frac{D}{2} = \frac{1}{2} = .05\right\}$$

$$= 3.50 \text{ m/s}$$

(*iii*) **Reynold number, R_e**

$$R_e = \frac{\bar{u} \times D}{\nu} = \frac{\bar{u} \times D}{\mu / \rho} = \frac{\rho \times \bar{u} \times D}{\mu}$$

$$= 1300 \times \frac{3.50 \times 0.1}{0.7} = \mathbf{650.00.\ Ans.}$$

Problem 9.6 *What power is required per kilometre of a line to overcome the viscous resistance to the flow of glycerine through a horizontal pipe of diameter 100 mm at the rate of 10 litres/s ? Take μ = 8 poise and kinematic viscosity (ν) = 6.0 stokes.* (Delhi University, 1982)

Solution. Given :

Length of pipe, $L = 1 \text{ km} = 1000 \text{ m}$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

Discharge, $Q = 10 \text{ litres/s} = \frac{10}{1000} \text{ m}^3\text{/s} = .01 \text{ m}^3\text{/s}$

Viscosity, $\mu = 8 \text{ poise} = \frac{8}{10} \frac{\text{Ns}}{\text{m}^2} = 0.8 \text{ N s/m}^2$

Kinematic Viscosity, $\nu = 6.0 \text{ stokes}$ $\left(\because \; 1 \text{ poise} = \frac{1}{10} \text{Ns / m}^2\right)$

$$= 6.0 \text{ cm}^2\text{/s} = 6.0 \times 10^{-4} \text{ m}^2\text{/s}$$

Loss of pressure head is given by equation (9.6) as $h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$

$$\text{Power required} = W \times h_f \text{ watts} \qquad \ldots(i)$$

where W = weight of oil flowing per sec = $\rho g \times Q$

Substituting the values of W and h_f in equation (*i*),

$$\text{Power required} = (\rho g \times Q) \times \frac{(32\,\mu\bar{u}L)}{\rho g D^2} \text{ watts} = \frac{Q \times 32\,\mu\bar{u}L}{D^2} \qquad \text{(cancelling } \rho g\text{)}$$

But $$\bar{u} = \frac{Q}{\text{Area}} = \frac{.01}{\frac{\pi}{4} D^2} = \frac{.01 \times 4}{\pi \times (.1)^2} = 1.273 \text{ m/s}$$

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or $$p_1 - p_2 = \frac{12\mu\bar{u}L}{t^2} \qquad [\because x_1 - x_2 = L]$$

If h_f is the drop of pressure head, then

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2} \qquad ...(9.13)$$

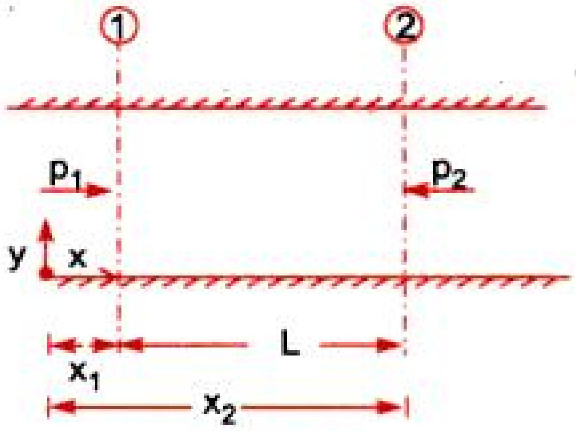

Fig. 9.8

(*iv*) **Shear Stress Distribution.** It is obtained by substituting the value of u from equation (9.9) into

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\therefore \qquad \tau = \mu \frac{\partial u}{\partial y} = u \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\right] = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y] \qquad ...(9.14)$$

In equation (9.14), $\frac{\partial p}{\partial x}$ and t are constant. Hence τ varies linearly with y. The shear stress distribution is shown in Fig. 9.7 (*b*). Shear stress is maximum, when $y = 0$ or t at the walls of the plates. Shear stress is zero, when $y = t/2$ that is at the centre line between the two plates. Max. shear stress (τ_0) is given by

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} t. \qquad ...(9.15)$$

Problem 9.7. *Calculate : (a) the pressure gradient along flow, (b) the average velocity, and (c) the discharge for an oil of viscosity 0.02 Ns/m² flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s.*

(Delhi University, 1982)

Solution. Given :

Viscosity, $\mu = .02$ N s/m^2
Width, $b = 1$ m
Distance between plates, $t = 10$ mm $= .01$ m
Velocity midway between the plates, $U_{max} = 2$ m/s.

(*i*) **Pressure gradient** $\left(\frac{dp}{dx}\right)$

Using equation (9.10), $U_{max} = -\frac{1}{8\mu}\frac{dp}{dx}t^2$ or $2.0 = -\frac{1}{8 \times .02}\left(\frac{dp}{dx}\right)(.01)^2$

$$\therefore \qquad \frac{dp}{dx} = -\frac{2.0 \times 8 \times .02}{.01 \times .01} = \mathbf{-3200\ N/m^2\ per\ m.\ Ans.}$$

(*ii*) **Average velocity** ($\bar{u}$)

Using equation (9.12), $\frac{U_{max}}{\bar{u}} = \frac{3}{2} \quad \therefore \quad \bar{u} = \frac{2U_{max}}{3} = \frac{2 \times 2}{3} =$ **1.33 m/s. Ans.**

(*iii*) **Discharge** (Q) $\qquad = \text{Area of flow} \times \bar{u} = b \times t \times \bar{u} = 1 \times .01 \times 1.33 =$ **.0133 m³/s. Ans.**

(*ii*) **Rate of leakage**, Q

$$Q = \bar{u} \times \text{area of flow}$$
$$= 0.193 \times \pi D \times t \text{ m}^3/\text{s} = 0.193 \times \pi \times .1 \times .0001 \text{ m}^3/\text{s}$$
$$= 6.06 \times 10^{-6} \text{ m}^3/\text{s} = 6.06 \times 10^{-6} \times 10^3 \text{ litre/s}$$
$$= \mathbf{6.06 \times 10^{-3} \text{ litre/s. Ans.}}$$

▶ 9.4 KINETIC ENERGY CORRECTION AND MOMENTUM CORRECTION FACTORS

Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section. It is denoted by α. Hence mathematically,

$$\alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}} \quad ...(9.16)$$

Momentum Correction Factor. It is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on average velocity across a section. It is denoted by β. Hence mathematically,

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}. \quad ...(9.17)$$

Problem 9.13 *Show that the momentum correction factor and energy correction factor for laminar flow through a circular pipe are 4/3 and 2.0 respectively.*

Solution. (*i*) **Momentum Correction Factor or β**

The velocity distribution through a circular pipe for laminar flow at any radius r is given by equation (9.3)

or
$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2) \quad ...(1)$$

Consider an elementary area dA in the form of a ring at a radius r and of width dr, then

$$dA = 2\pi r\, dr$$

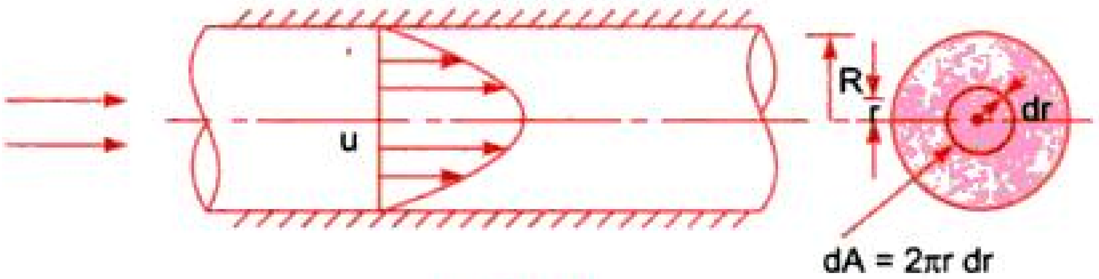

Fig. 9.9

Rate of fluid flowing through the ring

$$= dQ = \text{velocity} \times \text{area of ring element}$$
$$= u \times 2\pi r\, dr$$

Momentum of the fluid through ring per second

$$= \text{mass} \times \text{velocity}$$
$$= \rho \times dQ \times u = \rho \times 2\pi r\, dr \times u \times u = 2\pi\rho\, u^2 r\, dr$$

∴ Total actual momentum of the fluid per second across the section

$$= \int_0^R 2\pi\rho\, u^2 r\, dr$$

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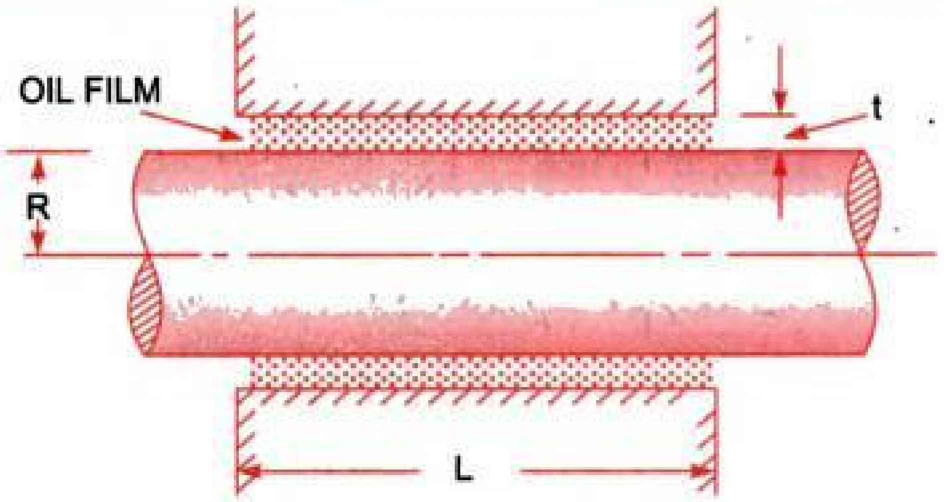

Fig. 9.10 *Journal bearing.*

As the thickness of oil film is very small, the velocity distribution in the oil film can be assumed as linear.

Hence $$\frac{du}{dy} = \frac{V-0}{t} = \frac{V}{t} = \frac{\pi DN}{60 \times t}$$

$\therefore$ $$\tau = \mu \frac{\pi DN}{60 \times t}$$

$\therefore$ Shear force or viscous resistance = $\tau \times$ Area of surface of shaft

$$= \frac{\mu \pi DN}{60t} \times \pi DL = \frac{\mu \pi^2 D^2 NL}{60t}$$

$\therefore$ Torque required to overcome the viscous resistance,

$$T = \text{Viscous resistance} \times \frac{D}{2}$$

$$= \frac{\mu \pi^2 D^2 NL}{60t} \times \frac{D}{2} = \frac{\mu \pi^2 D^3 NL}{120t}$$

$\therefore$ Power absorbed in overcoming the viscous resistance

$$*P = \frac{2\pi NT}{60} = \frac{2\pi N}{60} \times \frac{\mu \pi^2 D^3 NL}{120t}$$

$$= \frac{\mu \pi^3 D^3 N^2 L}{60 \times 60 \times t} \text{ watts. Ans.} \quad \text{...(9.18)}$$

Problem 9.14 *A shaft having a diameter of 50 mm rotates centrally in a journal bearing having a diameter of 50.15 mm and length 100 mm. The angular space between the shaft and the bearing is filled with oil having viscosity of 0.9 poise. Determine the power absorbed in the bearing when the speed of rotation is 60 r.p.m.*

Solution. Given :

Dia. of shaft, $D = 50$ mm or .05 m

Dia. of bearing, $D_1 = 50.15$ mm or 0.05015 m

Length, $L = 100$ mm or 0.1 m

*Power, $P = T \times \omega = T \times \frac{2\pi N}{60} = \frac{2\pi NT}{60}$ watts $= \frac{2\pi NT}{60{,}000}$ kW.

Problem 9.19 *Find the torque required to rotate a vertical shaft of diameter 100 mm at 750 r.p.m. The lower end of the shaft rests in a foot-step bearing. The end of the shaft and surface of the bearing are both flat and are separated by an oil film of thickness 0.5 mm. The viscosity of the oil is given 1.5 poise.* (A.M.I.E., Summer, 1977)

Solution. Given :

Dia. of shaft, $D = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ $R = \frac{D}{2} = \frac{0.1}{2} = 0.05 \text{ m}$

$N = 750 \text{ r.p.m.}$

Thickness of oil film, $t = 0.5 \text{ mm} = 0.0005 \text{ m}$

$\mu = 1.5 \text{ poise} = \frac{1.5}{10} \frac{\text{Ns}}{\text{m}^2}$

The torque required is given by equation (9.19) or

$$T = \frac{\mu}{60t} \pi^2 N R^4 \text{ Nm}$$

$$= \frac{1.5}{10} \times \frac{\pi^2 \times 750 \times (.05)^4}{60 \times .0005} = \mathbf{0.2305 \text{ Nm. Ans.}}$$

Problem 9.20 *Find the power required to rotate a circular disc of diameter 200 mm at 1000 r.p.m. The circular disc has a clearance of 0.4 mm from the bottom flat plate and the clearance contains oil of viscosity 1.05 poise.*

Solution. Given :

Dia. of disc, $D = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ $R = \frac{D}{2} = \frac{0.2}{2} = 0.1 \text{ m}$

$N = 1000 \text{ r.p.m.}$

Thickness of oil film, $t = 0.4 \text{ mm} = 0.0004 \text{ m}$

$\mu = 1.05 \text{ poise} = \frac{1.05}{10} \text{ N s/m}^2$

The power required to rotate the disc is given by equation (9.20) or

$$P = \frac{\mu \pi^3 N^2 R^4}{60 \times 30 \times t} \text{ watts}$$

$$= \frac{1.05}{10} \times \frac{\pi^3 \times 1000^2 \times (0.1)^4}{60 \times 30 \times .0004} = \mathbf{452.1 \text{ W. Ans.}}$$

9.5.3 Viscous Resistance of Collar Bearing.

Fig. 9.12 shows the collar bearing, where the face of the collar is separated from bearing surface by an oil film of uniform thickness.

Let N = Speed of the shaft in r.p.m.

R_1 = Internal radius of the collar

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Problem 9.24 *A pipe of diameter 20 cm and length 10^4 m is laid at a slope of 1 in 200. An oil of sp. gr. 0.9 and viscosity 1.5 poise is pumped up at the rate of 20 litres per second. Find the head lost due to friction. Also calculate the power required to pump the oil.*

Solution. Given :

Dia. of pipe, $D = 20 \text{ cm} = 2.50 \text{ m}$

Length of pipe, $L = 10000 \text{ m}$

Slope of pipe, $i = 1 \text{ in } 200 = \frac{1}{200}$

Sp. gr. of oil, $S = 0.9$

$\therefore$ Density of oil, $\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Viscosity of oil, $\mu = 1.5 \text{ poise} = \frac{1.5}{10} \frac{\text{Ns}}{\text{m}^2}$

Discharge, $Q = 20 \text{ litre/s} = 0.02 \text{ m}^3\text{/s}$ $\{\because \; 1000 \text{ litres} = 1 \text{ m}^3\}$

$\therefore$ Velocity of flow, $\bar{u} = \frac{Q}{\text{Area}} = \frac{0.020}{\frac{\pi}{4} D^2} = \frac{0.020}{\frac{\pi}{4} (.2)^2} = 0.6366 \text{ m/s}$

$\therefore$ $R_e = \text{Reynold number}$

$$= \frac{\rho V D}{\mu} = \frac{900 \times 0.6366 \times .2}{\frac{1.5}{10}}$$

$$= \frac{900 \times .6366 \times .2 \times 10}{1.5} \qquad \{\because \; V = \bar{u} = 0.6366\}$$

$$= 763.89$$

As the Reynold number is less than 2000, the flow is viscous. The co-efficient of friction for viscous flow is given by equation (9.23) as

$$f = \frac{16}{R_e} = \frac{16}{763.89} = 0.02094$$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \,.\, f \,.\, L \,.\, \bar{u}^2}{D \times 2g}$

$$= \frac{4 \times .02094 \times 10000 \times (.6366)^2}{0.2 \times 2 \times 9.81} \text{ m} = \textbf{86.50 m. Ans.}$$

Due to slope of pipe 1 in 200, the height through which oil is to be raised by pump

$$= \text{Slope} \times \text{Length of pipe}$$

$$= i \times L = \frac{1}{200} \times 10000 = 50 \text{ m}$$

$\therefore$ Total head against which pump is to work,

$$H = h_f + i \times L = 86.50 + 50 = 136.50 \text{ m}$$

$\therefore$ Power required to pump the oil

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and μ = Coefficient of viscosity.

Using Hagen Poiseuilli's Formula, $h = \dfrac{32\mu\bar{u}L}{\rho g D^2}$

But $$\bar{u} = \frac{Q}{\text{Area}} = \frac{Q}{\frac{\pi}{4}D^2}$$

where Q is rate of liquid flowing through tube.

$$h = \frac{32\mu \times \dfrac{D}{\frac{\pi}{4}D^2} \times L}{\rho g D^2} = \frac{128\,\mu\, Q\,.\,L}{\pi \rho g D^4}$$

or $$\mu = \frac{\pi \rho g h \overline{D}}{128\, Q\,.\,L} \qquad ...(9.25)$$

Measurement of D should be done very accurately.

9.8.2 Falling Sphere Resistance Method.

Theory. This method is based on Stoke's law, according to which the drag force, F on a small sphere moving with a constant velocity, U through a viscous fluid of viscosity, μ for viscous conditions is given by

$$F = 3\pi\mu U d \qquad ...(i)$$

where d = diameter of sphere

U = velocity of sphere.

When the sphere attains a constant velocity U, the drag force is the difference of between the weight of sphere and buoyant force acting on it.

Let L = distance travelled by sphere in viscous fluid,

t = time taken by sphere to cover distance l,

ρ_s = density of sphere,

ρ_f = density of fluid,

W = weight of sphere,

and F_B = buoyant force acting on sphere.

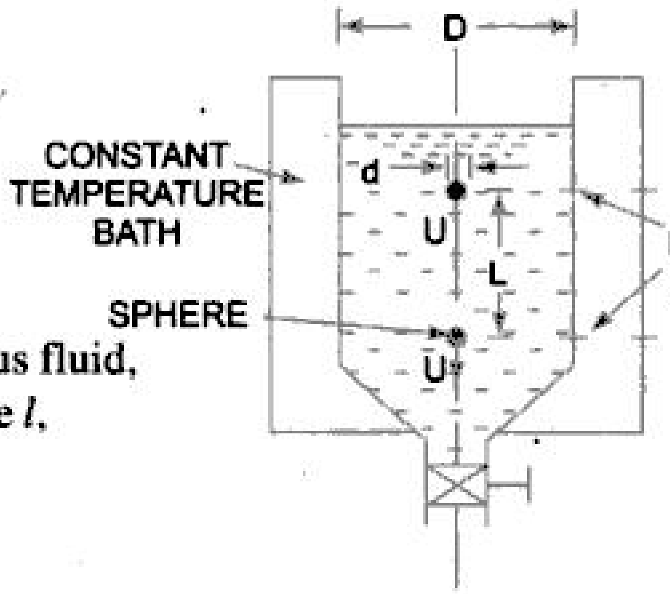

Fig. 9.15 *Falling sphere resistance method.*

Then constant velocity of sphere, $U = \dfrac{L}{t}$

Weight of sphere, W = volume × density of sphere × g

$$= \frac{\pi}{6} d^3 \times \rho_s \times g \qquad \left\{\because \text{ volume of sphere} = \frac{\pi}{6} d^3\right\}$$

and buoyant force, F_B = weight of fluid displaced

= volume of liquid displaced × density of fluid × g

$$= \frac{\pi}{6} d^3 \times \rho_t \times g \qquad \{\text{volume of liquid displaced} = \text{volume of sphere}\}$$

For equilibrium,

Viscosity, $\mu = 0.25$ poise

$$= \frac{0.25}{10} \text{ Ns/m}^2$$

Let the rate of flow of liquid= Q

Using equation (9.25), we get $\mu = \dfrac{\pi\rho g h D^4}{128 . Q . L} = \pi\rho g \times \dfrac{\dfrac{0.6867 \times 10^4}{\rho g} \times \left(2 \times 10^{-3}\right)^4}{128 \times Q \times 0.1}$

or
$$\frac{0.25}{10} = \frac{\pi \times 0.6867 \times 10^4 \times \left(2 \times 10^{-3}\right)^4}{128 \times Q \times 0.1}$$

or
$$Q = \frac{\pi \times 0.6867 \times 10^4 \times 2^4 \times 10^{-12} \times 10}{128 \times 0.1 \times 0.25} \text{ m}^3/\text{s}$$

$$= 107.86 \times 10^{-8} \text{ m}^3/\text{s} = 107.86 \times 10^{-8} \times 10^6 \text{ cm}^3/\text{s}$$

$$= 107.86 \times 10^{-2} \text{ cm}^3/\text{s} = \mathbf{1.078\ cm^3/s.\ Ans.}$$

Problem 9.28 *A sphere of diameter 2 mm falls 150 mm in 20 seconds in a viscous liquid. The density of the sphere is 7500 kg/m³ and of liquid is 900 kg/m³. Find the co-efficient of viscosity of the liquid.*

Solution. Given :

Dia. of sphere, $d = 2 \text{ mm} = 2 \times 10^{-3}$ m

Distance travelled by sphere $= 150$ mm $= 0.15$ m

Time taken, $t = 20$ seconds

Velocity of sphere, $U = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{0.15}{20} = .0075$ m/s

Density of sphere, $\rho_s = 7500 \text{ kg/m}^3$

Density of liquid, $\rho_f = 900 \text{ kg/m}^3$

Using relation (9.26), we get $\mu = \dfrac{gd^2}{18U}[\rho_s - \rho_f] = \dfrac{9.81 \times \left[2 \times 10^{-3}\right]^2}{18 \times 0.0075}[7500 - 900]$

$$= \frac{9.81 \times 4 \times 10^{-6} \times 6600}{18 \times 0.0075} = 1.917 \frac{\text{Ns}}{\text{m}^2}$$

$$= 1.917 \times 10 = \mathbf{19.17\ poise.\ Ans.}$$

Problem 9.29 *Find the viscosity of a liquid of sp. gr. 0.8, when a gas bubble of diameter 10 mm rises steadily through the liquid at a velocity of 1.2 cm/s. Neglect the weight of the bubble.*

Solution. Given :

Sp. gr. of liquid $= 0.8$

$\therefore$ Density of liquid, $\rho_f = 0.8 \times 1000 = 800 \text{ kg/m}^3$

Dia. of gas bubble, $D = 10 \text{ mm} = 1 \text{ cm} = 0.01$ m

Velocity of bubble, $U = 1.2 \text{ cm/s} = .012$ m/s

As weight of bubble is neglected and density of bubble

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4. The kinetic energy correction factor α is given as

$$\alpha = \frac{\text{K.E. per second based on actual velocity}}{\text{K.E. per second based on average velocity}}$$

$= 2.0$ ...... for a circular pipe.

5. Momentum correction factor, β is given by

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}$$

$= \frac{4}{3}$ ...... for a circular pipe.

6. For the viscous resistance of Journal Bearing.

$$V = \frac{\pi DN}{60}, \frac{du}{dy} = \frac{V}{t} = \frac{\pi DN}{60t}$$

$$\tau = \frac{\mu\pi dN}{660t}, \text{ Shear force} = \frac{\mu\pi^2 D^2 NL}{60t}$$

Torque, $T = \frac{\mu\pi^2 D^3 NL}{120t}$ and power $= \frac{\mu\pi^3 D^3 N^2 L}{60 \times 60 \times t}$

where L= length of bearing, N = speed of shaft

t = clearance between the shaft and bearing.

7. For the Foot-Step Bearing, the shear force, torque and h.p. absorbed are given as :

Shear force, $F = \frac{\mu}{15} \frac{\pi^2 N}{t} \frac{R^3}{3}$

Torque, $T = \frac{\mu}{60t} \pi^2 NR^4$

Power $= \frac{\mu\pi^3 N^2 R^4}{60 \times 30 \times t}$

where R = radius of the shaft, N = speed of the shaft.

8. For the collar bearing the torque and power absorbed are given as

$$T = \frac{\mu}{60t} \pi^2 N [R_2{}^4 - R_1{}^4], \qquad P = \frac{\mu\pi^3 N^2}{60 \times 30t} [R_2{}^4 - R_1{}^4]$$

where R_1 = internal radius of the collar, R_2 = external radius of the collar,

t = thickness of oil film, P = power in watts.

9. For the viscous flow the co-efficient of friction is given by, $f = \frac{16}{R_e}$

where R_e = the Reynold number $= \frac{\rho VD}{\mu} = \frac{VD}{\nu}$.

10. The co-efficient of viscosity is determined by dash-pot arrangement as $\mu = \frac{4Wt^3}{3\pi LD^3 V}$

where W = weight of the piston, t = clearance between dash-pot and piston,

L = length of the piston, D = diameter of the piston,

V = velocity of the piston.

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$$\mu = \frac{4Wt^3}{3\pi D^3 LV} = \frac{4 \times 10 \times 0.002^3}{3\pi \times 0.1^3 \times 0.15 \times .45} = 503 \times 10^{-6} \text{ N s/m}^2 \quad]$$

20. A liquid is pumped through a 15 cm diameter and 300 m long pipe at the rate of 20 tonnes per hour. The density of liquid is 910 kg/m^3 and kinematic viscosity = 0.002 m^2/s. Determine the power required and show that the flow is viscous.

[**Hint.** D = 15 cm = 0.15 m, L = 300 m, W = 20 tonnes/hr

= 20 × 1000 kgf/60 × 60 sec = 5.555 kgf/sec = 5.555 × 9.81 N/s.

$$Q = \frac{W}{\rho g} = \frac{5.555 \times 9.81}{910 \times 9.81} = 0.0061 \text{ m}^3\text{/s. } V = \frac{Q}{A} = \frac{0.0061}{\frac{\pi}{4}(.15^2)}$$

= 0.345 m/s, ν = 0.002 m^2/s.

Now $$R_e = \frac{\rho VD}{\mu} = \frac{V \times D}{\nu} = \frac{0.345 \times 0.15}{0.002} = 25.87$$

which is less than 2000. Hence flow is viscous.

$$h_f = 32\ \mu LV/\rho g D^2, \text{ where } \nu = \frac{\mu}{\rho} \quad \therefore \mu = \nu \times \rho = 0.002 \times 910 = 1.82$$

Hence, $$h_f = \frac{32 \times 1.82 \times 300 \times 0.345}{(910 \times 9.81 \times 0.15^2)} = 30$$

∴ $P = \rho g.Q.h_f/1000 = 910 \times 9.81 \times 0.0061 \times 30/1000 =$ **1.633 kW. Ans.]**

21. An oil of specific gravity 0.9 and viscosity 10 poise is flowing through a pipe of diameter 110 mm. The velocity at the centre is 2 m/s, find : (*i*) pressure gradient in the direction of flow, (*ii*) shear stress at the pipe wall ; (*iii*) Reynolds numbers, and (*iv*) velocity at a distance of 30 mm from the wall.

(Delhi University, 1996)

[**Hint.** ρ = 900 kg/m^3 ; μ = 10 poise = 1 N s/m^2 ; D = 110 mm = 0.11 m,

$$U_{max} = 2 \text{ m/s ; } \bar{u} = 1 \text{ m/s ; } U_{max} = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right) R^2$$

(*i*) $$\left(\frac{-dp}{dx}\right) = \frac{4\mu \times U_{max}}{R^2} = \frac{4 \times 1 \times 2}{0.055^2} = 2644.6 \text{ N/m}^3 ;$$

(*ii*) $$\tau_0 = \left(\frac{-dp}{dx}\right) \times \frac{R}{2} = 2644.6 \times \frac{0.055}{2} = 72.72 \text{ N/m}^2 ;$$

(*iii*) $$R_e = \frac{\rho \times \bar{u} \times D}{\mu} = \frac{900 \times 1 \times 0.11}{1} = 99 \text{ ; and}$$

(*iv*) $$u = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)(R^2 - r^2) = \frac{1}{4 \times 1}(2644.6)(0.055^2 - 0.025^2) = 1.586 \text{ m/s.]}$$

22. Determine (*i*) the pressure gradient, (*ii*) the shear stress at the two horizontal plates, (*iii*) the discharge per metre width for laminar flow of oil with a maximum velocity of 2 m/s between two plates which are 150 mm apart. Given : μ = 2.5 N s/m^2. *(Delhi University, December 2002)*

[**Hint.** U_{max} = 2 m/s, t = 150 mm = 0.15 m, μ = 2.5 N s/m^2

(*i*) $$U_{max} = -\frac{1}{8\mu}\frac{dp}{dx}t^2 \quad \therefore \quad \frac{dp}{dx} = \frac{-8\mu\, U_{max}}{t^2} = \frac{-8 \times 2.5 \times 2}{0.15^2} = \mathbf{-1777.77\ N/m^2.\ Ans.}$$

(*ii*) $$\tau_0 = -\frac{1}{2}\frac{dp}{dx} \times t = -\frac{1}{2}(-1777.77) \times 0.15 = \mathbf{133.33\ N/m^2.\ Ans.}$$

(*iii*) $$Q = \text{Mean velocity} \times \text{Area} = \left(\frac{2}{3}U_{max}\right) \times (t \times 1) = \left(\frac{2}{3} \times 2\right) \times (0.15 \times 1) = \mathbf{0.2\ m^3/s.\ Ans.]}$$

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or $$\log_{10}(y/R) = -\frac{3.75}{5.75} = -0.6521 = -\bar{1}.3479$$

$\therefore$ $$y/R = 0.22279 \simeq 0.2228 \text{ or } y = \mathbf{.2228\ R.\ Ans.}$$

Problem 10.6 *For turbulent flow in a pipe of diameter 300 mm, find the discharge when the centre-line velocity is 2.0 m/s and the velocity at a point 100 mm from the centre as measured by pitot-tube is 1.6 m/s.*

Solution. Given :

Dia. of pipe, $D = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Radius, $R = \frac{0.3}{2} = 0.15 \text{ m}$

Velocity at centre, $u_{max} = 2.0 \text{ m/s}$

Velocity (at $r = 100 \text{ mm} = 0.1 \text{ m}$), $u = 1.6 \text{ m/s}$

Now $y = R - r = 0.15 - 0.10 = 0.05 \text{ m}$

$\therefore$ Velocity (at $r = 0.1$ m or at $y = 0.05$ m), $u = 1.6$ m/s

The velocity in terms of centre-line velocity is given by equation (10.18) as

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10}(R/y)$$

Substituting the values, we get $\frac{2.0 - 1.6}{u_*} = 5.75 \log_{10} \frac{.15}{.05}$ $\left[\because \begin{array}{l} y = .05 \text{ m} \\ R = 0.15 \text{ m} \end{array}\right]$

$$= 5.75 \log_{10} 3.0 = 2.7434$$

or $$\frac{0.4}{u_*} = 2.7434$$

$\therefore$ $$u_* = \frac{0.4}{2.743} = 0.1458 \text{ m/s} \quad ...(i)$$

Using equation (10.26) which gives relation between velocity at any point and average velocity, we have

$$\frac{u - \overline{U}}{u_*} = 5.75 \log_{10}(y/R) + 3.75$$

at $y = R$, velocity u becomes $= u_{max}$

$\therefore$ $$\frac{u_{max} - \overline{U}}{u_*} = 5.75 \log_{10}(R/R) + 3.75 = 5.75 \times 0 + 3.75 = 3.75$$

But $u_{max} = 2.0$ and u_* from (*i*) $= 0.1458$

$\therefore$ $$\frac{2.0 - \overline{U}}{0.1458} = 3.75$$

or $$\overline{U} = 2.0 - .1458 \times 3.75 = 2.0 - 0.5467 = 0.4533 \text{ m/s}$$

$\therefore$ Discharge, $Q = \text{Area} \times \text{average velocity}$

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11 CHAPTER

Flow Through Pipes

▶ 11.1 INTRODUCTION

In the chapters 9 and 10, laminar flow and turbulent flow have been discussed. We have seen that when the Reynold number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynold number is more than 4000, the flow is known as turbulent flow. In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full as in the case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channels. Here we will consider flow of fluids through pipes under pressure only.

▶ 11.2 LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Engergy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :
   (*a*) Dracy-Weisbach Formula
   (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to
   (*a*) Sudden expansion of pipe
   (*b*) Sudden contraction of pipe
   (*c*) Bend in pipe
   (*d*) Pipe fittings etc.
   (*e*) An obstruction in pipe.

▶ 11.3 LOSS OF ENERGY (OR HEAD) DUE TO FRICTION

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 \, . \, f \, . \, L \, . \, V^2}{d \times 2g} \qquad \text{...(11.1)}$$

where h_f = loss of head due to friction

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$\therefore$ Head lost due to friction, $h_f = \dfrac{4 \, . \, f \, . \, L \, . \, V^2}{d \times 2g} = \dfrac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81}$ = **3.61 m. Ans.**

Problem 11.4 *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take ν = .29 stokes.*

Solution. Given :

Sp. gr. of oil, $S = 0.7$

Dia. of pipe, $d = 300$ mm $= 0.3$ m

Discharge, $Q = 500$ litres/s $= 0.5$ m^3/s

Length of pipe, $L = 1000$ m

Velocity, $V = \dfrac{Q}{\text{Area}} = \dfrac{0.5}{\frac{\pi}{4} d^2} = \dfrac{0.5 \times 4}{\pi \times 0.3^2} = 7.073$ m/s

$\therefore$ Reynold number, $R_e = \dfrac{V \times d}{\nu} = \dfrac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times (10)^4$

$\therefore$ Co-efficient of friction, $f = \dfrac{.079}{R_e^{1/4}} = \dfrac{0.79}{\left(7.316 \times 10^4\right)^{1/4}} = .0048$

$\therefore$ Head lost due to friction, $h_f = \dfrac{4 \times f \times L \times V^2}{d \times 2g} = \dfrac{4 \times .0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18$ m

Power required $= \dfrac{\rho g \, . \, Q \, . \, h_f}{1000}$ kW

where ρ = density of oil $= 0.7 \times 1000 = 700$ kg/m^3

$\therefore$ Power required $= \dfrac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$ = **560.28 kW. Ans.**

Problem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \dfrac{4 \, . \, f \, . \, L \, . \, V^2}{d \times 2g}$.

Solution. Given :

Dia. of pipe, $d = 200$ mm $= 0.20$ m

Length of pipe, $L = 500$ m

Difference of pressure head, $h_f = 4$ m of water

$f = .009$

Using equation (11.1), we have $h_f = \dfrac{4 \times f \times L \times V^2}{d \times 2g}$

or $4.0 = \dfrac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81}$ or $V^2 = \dfrac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$

Substituting the value of $\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right)$ in equation (i), we get

$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} = \left(\frac{V_1 - V_2}{2g}\right)^3$$

$$\therefore \qquad h_e = \frac{(V_1 - V_2)^2}{2g}. \qquad \text{...(11.5)}$$

11.4.2 Loss of Head due to Sudden Contraction. Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in Fig. 11.2. Consider two sections 1-1 and 2-2 before and after, contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section C-C as shown in Fig. 11.2. This section C-C is called Vena-contracta. After section C-C a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

Let A_c = Area of flow at section C-C

V_c = Velocity of flow at section C-C

A_2 = Area of flow at section 2-2

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction.

Now h_c = actually loss of head due to enlargement from section C-C to section 2-2 and is given by equation (11.5) as

$$= \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \qquad \text{...}(i)$$

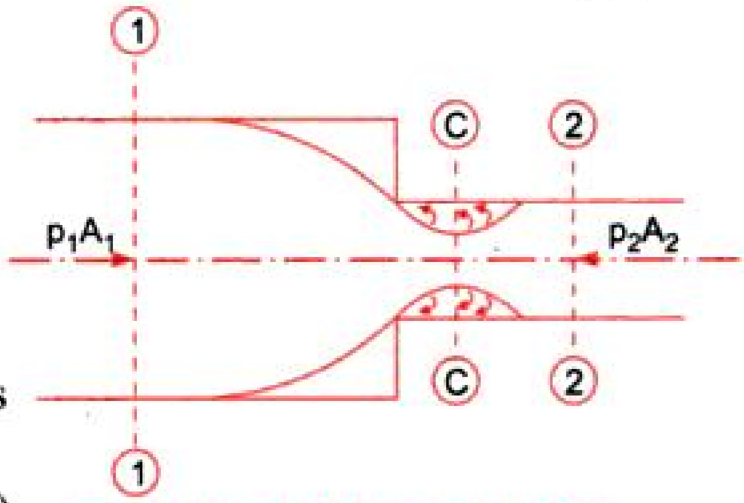

Fig. 11.2 *Sudden contraction.*

From continuity equation, we have

$$A_cV_c = A_2V_2 \quad \text{or} \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c} \qquad \left[\because V_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in (i), we get

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 \qquad \text{...(11.6)}$$

$$= \frac{kV_2^2}{2g}, \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

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Dia. of smaller pipe, $D_2 = 250 \text{ mm} = 0.25 \text{ m}$

$\therefore$ Area, $A_2 = \dfrac{\pi}{4} (.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

Pressure in smaller pipe, $p_2 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

$C_c = 0.62$

Head lost due to contraction $= \dfrac{V_2^2}{2g}\left[\dfrac{1}{C_c} - 1.0\right]^2 = \dfrac{V_2^2}{2g}\left[\dfrac{1}{0.62} - 1.0\right]^2 = 0.375 \dfrac{V_2^2}{2g}$

From continuity equation, we have $A_1V_1 = A_2V_2$

or
$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V^2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (pipe is horizontal)

$\therefore$
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

But $h_c = 0.375 \dfrac{V_2^2}{2g}$ and $V_1 = \dfrac{V_2}{4}$

Substituting these values in the above equation, we get

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(V_2/4)^2}{2g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{V_2^2}{2g} + 0.375 \frac{V_2^2}{2g}$$

or
$$14.0 + \frac{V_2^2}{16 \times 2g} = 12.0 + 1.375 \frac{V_2^2}{2g}$$

or
$$14 - 12 = 1.375 \frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.3125 \frac{V_2^2}{2g}$$

or
$$2.0 = 1.3125 \times \frac{V_2^2}{2g} \text{ or } V_2 = \sqrt{\frac{2.0 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375 \dfrac{V_2^2}{2g} = \dfrac{0.375 \times (5.467)^2}{2 \times 9.81}$ = **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 5.467 = 0.2683 \text{ m}^3\text{/s}$ = **268.3 lit/s. Ans.**

Problem 11.12 *If in the problem 11.11, the rate of flow of water is 300 litres/s, other data remaining the same, find the value of co-efficient of contraction, C_c.*

But $$Z_1 = Z_2 \text{ (as pipe is horizontal)}$$

$\therefore$ $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Substituting the values of p_1, p_2, h_c and V_1, we get

$$\frac{103005}{1000 \times 9.81} + \frac{(V_2/4)^2}{2g} = \frac{67689}{1000 \times 9.81} + \frac{V_2^2}{2g} + .2899\,\frac{V_2^2}{2g}$$

or $$10.5 + \frac{V_2^2}{16 \times 2g} = 6.9 + 1.2899\,\frac{V_2^2}{2g}$$

or $$10.5 - 6.9 = 1.2899\,\frac{V_2^2}{2g} - \frac{1}{16} \times \frac{V_2^2}{2g} = 1.2274\,\frac{V_2^2}{2g}$$

or $$3.6 = 1.2274 \times \frac{V_2^2}{2g}$$

$\therefore$ $$V_2 = \sqrt{\frac{3.6 \times 2 \times 9.81}{1.2274}} = 7.586 \text{ m/s}$$

(*i*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 7.586$

$= \mathbf{0.3723\ m^3/s\ or\ 372.3\ lit/s.\ Ans.}$

(*ii*) Applying Bernoulli's equation to sections 3-3 and 4-4,

$$\frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 = \frac{p_4}{\rho g} + \frac{V_4^2}{2g} + Z_4 + \text{head loss due to sudden enlargement } (h_e)$$

But $$p_3 = 6900 \text{ kg/m}^2\text{, or } 67689 \text{ N/m}^2$$
$$V_3 = V_2 = 7.586 \text{ m/s}$$
$$V_4 = V_1 = \frac{V_2}{4} = \frac{7.586}{4} = 1.8965$$
$$Z_3 = Z_4$$

And head loss due to sudden enlargement is given by equation (11.5) as

$$h_e = \frac{(V_3 - V_4)^2}{2g} = \frac{(7.586 - 1.8965)^2}{2 \times 9.81} = 1.65 \text{ m}$$

Substituting these values in Bernoulli's equation, we get

$$\frac{67689}{1000 \times 9.81} + \frac{7.586^2}{2 \times 9.81} = \frac{p_4}{1000 \times 9.81} + \frac{1.8965^2}{2 \times 9.81} + 1.65$$

or $$6.9 + 2.933 = \frac{p_4}{1000 \times 9.81} + 0.183 + 1.65$$

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

Substituting these values, we have

$$4.0 = \frac{V^2}{g} + \frac{0.5\,V^2}{2g} + \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}\ [1.0 + 0.5 + 9.0]$$

$$= 10.5 \times \frac{V^2}{2g}$$

$\therefore$ $$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3/\text{s}$$

$$= \mathbf{85.89 \text{ litres/s. Ans.}}$$

Problem 11.17 *A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = .01 for both sections of the pipe.*

(Osmania University, 1992 ; A.M.I.E., Summer, 1978)

Solution. Given :

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Dia. of 1st pipe, $d_1 = 150$ mm $= 0.15$ m

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in Fig. 11.5 and taking reference line passing through the centre of pipe.

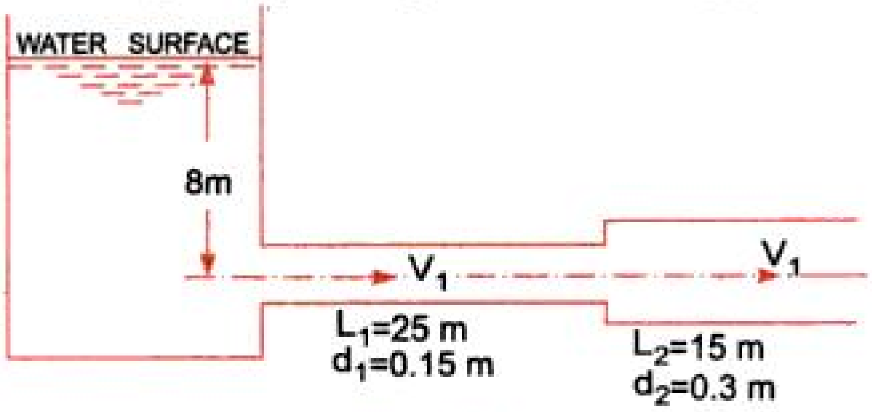

Fig. 11.5

$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

$$\therefore \qquad V = \sqrt{\frac{60}{3.639}} = 2.344 \text{ m/s}$$

$\therefore$ Discharge, $\qquad Q_2 = V \times \text{Area} = 2.344 \times \frac{\pi}{4} \times 1^2 = 1.841 \text{ m}^3/\text{s}$

percentage increase in the discharge $= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{(1.841 - 1.645)}{1.645} \times 100 =$ **11.91%. Ans.**

Problem 11.20 *Design the diameter of a steel pipe to carry water having kinematic viscosity $\nu = 10^{-6}$ m²/s with a mean velocity of 1 m/s. The head loss is to be limited to 5 m per 100 m length of pipe. Consider the equivalent sand roughness height of pipe $k_s = 45 \times 10^{-4}$ cm. Assume that the Darcy Weisbach friction factor over the whole range of turbulent flow can be expressed as*

$$f = 0.0055 \left[1 + \left(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e} \right)^{1/3} \right]$$

where D = Diameter of pipe and R_e = Reynolds number. (A.M.I.E., Summer, 1985)

Solution. Given :

Kinematic viscosity, $\quad \nu = 10^{-6}$ m²/s

Mean velocity, $\quad V = 1$ m/s

Head loss, $\quad h_f = 5$ m in a length $L = 100$ m

Value of $\quad k_s = 45 \times 10^{-4}$ cm $= 45 \times 10^{-6}$ m

Value of
$$f = 0.0055 \left[1 + \left(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e} \right)^{1/3} \right] \qquad \ldots(i)$$

Using Darcy Weisbach equation, $h_f = \frac{4 \times f \times L \times V^2}{D \times 2g}$

or
$$f = \frac{h_f \times D \times 2g}{4 \times L \times V^2} = \frac{5 \times D \times 2 \times 9.81}{4 \times 100 \times 1^2} = 0.2452\, D$$

Now the Reynolds number is given by,

$$R_e = \frac{\rho V D}{\mu} = \frac{V \times D}{\nu} \qquad \left(\because \nu = \frac{\mu}{\rho} \right)$$

$$= \frac{1 \times D}{10^{-6}} = 10^6\, D$$

Substituting the values of f, R_e and k_s in equation (i), we get

$$0.2452\, D = 0.0055 \left[1 + \left(20 \times 10^3 \times \frac{45 \times 10^{-6}}{D} + \frac{10^6}{10^6 D} \right)^{1/3} \right]$$

or
$$\frac{0.2452}{0.0055} D = \left[1 + \left(\frac{0.9}{D} + \frac{1}{D} \right)^{1/3} \right]$$

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{4 \times .01 \times 15 \times (1.113)^2}{0.3 \times 2 \times 9.81} = 0.126 \text{ m}$$

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063 \text{ m}$$

Also $$V_1^2/2g = \frac{4.452^2}{2 \times 9.81} = 1.0 \text{ m.}$$

Total Energy Line

(*i*) Point *A* lies on free surface of water.

(*ii*) Take $AB = h_i = 0.5$ m.

(*iii*) From *B*, draw a horizontal line. Take *BL* equal to the length of pipe, *i.e.*, L_1. From *L* draw a vertical line downward.

(*iv*) Cut the line $LC = h_{f_1} = 6.73$ m.

(*v*) Join the point *B* to *C*. From *C*, take a line *CD* vertically downward equal to $h_e = 0.568$ m.

(*vi*) From *D*, draw *DM* horizontal and from point *F* which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at *M*. From *M*, take a distance $ME = h_{f_2} = 0.126$. Join *DE*.

Then line *ABCDE* represents the total energy line.

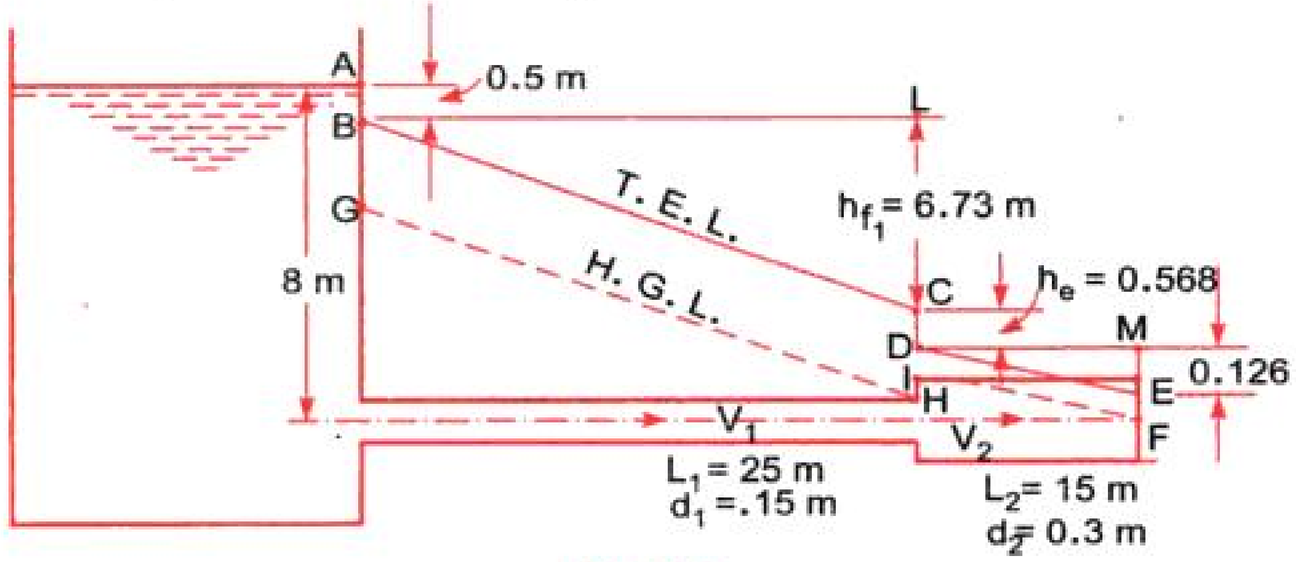

Fig. 11.9

Hydraulic Gradient Line (H.G.L.)

(*i*) From *B*, take $BG = \frac{V_1^2}{2g} = 1.0$ m.

(*ii*) Draw the line *GH* parallel to the line *BC*.

(*iii*) From *F*, draw a line *FI* parallel to the line *ED*.

(*iv*) Join the point *H* and *I*.

Then the line *GHIF* represents the hydraulic gradient line (H.G.L.).

Problem 11.24 *For the problem 11.18, draw the hydraulic gradient and total energy line.*

Solution. Refer to problem, 11.18,

Given: $d = 300 \text{ mm} = 0.3 \text{ m}$

$L = 400 \text{ m}, Q = 300 \text{ litres/s} = 0.3 \text{ m}^3\text{/s}$

$f = .008$

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.3 \text{ m}$

Length of pipe, $L = 3200 \text{ m}$

Mass, $M = 50 \text{ kg/s} = \rho \, . \, Q$

$\therefore$ Discharge, $$Q = \frac{50}{\rho} = \frac{50}{950} = 0.0526 \text{ m}^3\text{/s}$$

$\because$ Density, $\rho = 950 \text{ kg/m}^3$

Kinematic viscosity, $\nu = 2.1 \text{ stokes} = 2.1 \text{ cm}^2\text{/s}$

$= 2.1 \times 10^{-4} \text{ m}^2\text{/s}$

Height of upper end $= 40 \text{ m}$

Pressure at upper end $= \text{atmospheric} = 0$

Reynold number, $$R_e = \frac{V \times d}{\nu}, \text{ where } V = \frac{\text{Discharg}}{\text{Area}} = \frac{0.0526}{\frac{\pi}{4}(0.3)^2} = 0.744 \text{ m/s}$$

$\therefore$ $$R_e = \frac{0.744 \times 0.30}{2.1 \times 10^{-4}} = 1062.8$$

$\therefore$ Co-efficient of friction, $$f = \frac{16}{R_e} = \frac{16}{1062.8} = 0.015$$

Head lost due to friction, $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

$$= \frac{4 \times 0.015 \times 3200 \times (0.744)^2}{0.3 \times 2 \times 9.81} = 18.05 \text{ m of oil}$$

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = 0$, $z_2 = 40$ m, $V_1 = V_2$ as diameter is same

$p_2 = 0$, $h_f = 18.05$ m

$\therefore$ Substituting these values, we have

$$\frac{p_1}{\rho g} = 40 + 18.05 = 58.05 \text{ m of oil}$$

$\therefore$ $$p_1 = 58.05 \times \rho g = 58.05 \times 950 \times 9.81 \qquad [\because \rho \text{ for oil} = 950]$$

$$= 540997 \text{ N/m}^2 = \frac{540997}{10^{-4}} \text{ N/cm}^2 = \mathbf{54.099 \text{ N/cm}^2}\text{. Ans.}$$

H.G.L. and T.E.L.

$$\frac{V^2}{2g} = \frac{(.744)^2}{2 \times 9.81} = 0.0282 \text{ m}$$

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Let, L_1, L_2, L_3 = length of pipes 1, 2 and 3 respectively
d_1, d_2, d_3 = diameter of pipes 1, 2, 3 respectively
V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3
f_1, f_2, f_3 = co-efficient of frictions for pipes 1, 2, 3
H = difference of water level in the two tanks.

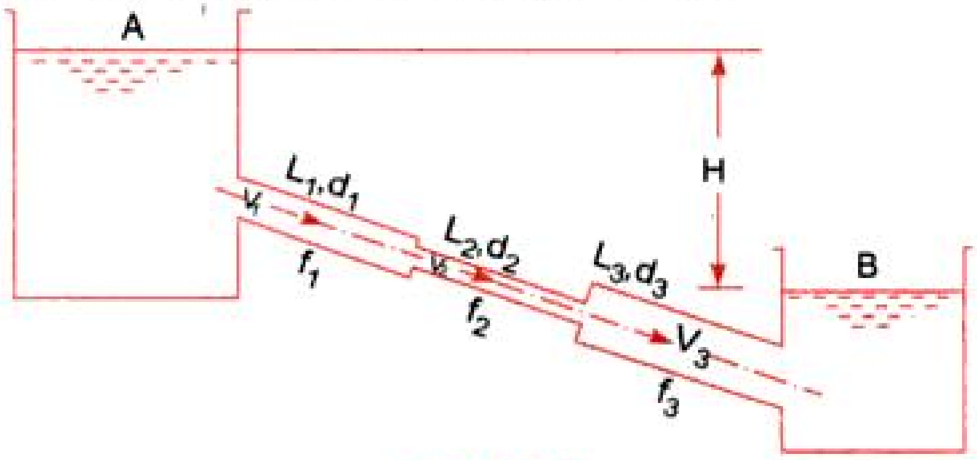

Fig. 11.16

The discharge passing through each pipe is same.

$\therefore$ $$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$\therefore$ $$I = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_2^2}{2g} \quad \text{...(11.12)}$$

If minor losses are neglected, then above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \quad \text{...(11.13)}$$

If the co-efficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then equation (11.13) becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right] \quad \text{...(11.14)}$$

Problem 11.30 *The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are .005, .0052 and .0048 respectively, considering : (i) minor losses also (ii) neglecting minor losses.*

(Delhi University, 1987)

Solution. Given :

Difference of water level, $H = 12$ m
Length of pipe 1, $L_1 = 300$ m and dia., $d_1 = 300$ mm $= 0.3$ m
Length of pipe 2, $L_2 = 170$ m and dia., $d_2 = 200$ mm $= 0.2$ m

▶ 11.8 EQUIVALENT PIPE

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1 and d_1 = dia.
L_2 = length of pipe 2 and d_2 = dia.
L_3 = length of pipe 3 and d_3 = dia.
H = total head loss
L = length of equivalent pipe
d = diameter of the equivalent pipe

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \qquad \text{...(11.14}A\text{)}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3 = \frac{\pi}{4} d_1^2V_1 = \frac{\pi}{4} d_2^2V_2 = \frac{\pi}{4} d_3^2V_3$

$\therefore$ $$V_1 = \frac{4Q}{\pi d_1^2},\ V_2 = \frac{4Q}{\pi d_2^2} \text{ and } V_3 = \frac{4Q}{\pi d_3^2}$$

Substituting these values in equation (11.14A), we have

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \qquad \text{...(11.15)}$$

Head loss in the equivalent pipe, $H = \frac{4f \, . \, L \, . \, V^2}{d \times 2g}$ [Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$\therefore$ $$H = \frac{4\,f\,.\,L\,.\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right] \qquad \text{...(11.16)}$$

Head loss in compound pipe and in equivalent pipe is same hence equating equations (11.15) and (11.16), we have

Let Q_1 = discharge in 1st parallel pipe

Q_2 = discharge in 2nd parallel pipe

$\therefore \quad Q = Q_1 + Q_2$

where Q = discharge in main pipe when pipes are parallel.

But as the length and diameters of each parallel pipe is same

$$\therefore \quad Q_1 = Q_2 = Q/2$$

Consider the flow through pipe ABC or ABD

Head loss through ABC = Head lost through AB + head lost through BC ...(*ii*)

But head lost due to friction through ABC = 0.3 m given

Head lost due to friction through $AB = \dfrac{4 \times f \times 750 \times V^2}{0.6 \times 2 \times 9.81}$, where V = velocity of flow through AB

$$= \frac{Q}{\text{Area}} = \frac{Q}{\frac{\pi}{4}(0.6)^2} = \frac{40}{\pi \times .36}$$

$\therefore$ Head lost due to friction through AB

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \left(\frac{4Q}{\pi \times .36}\right)^2 = 31.87\ Q^2$$

Head lost due to friction through BC

$$= \frac{4 \times f \times L_1 \times V_1^2}{d \times 2g}$$

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \left[\frac{Q}{2 \times \frac{\pi}{4}(.6)^2}\right] \quad \left[\because V_1 = \frac{\text{Distance}}{\frac{\pi}{4}(.6)^2} = \frac{Q}{2 \times \frac{\pi}{4} \times (.6)^2}\right]$$

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \frac{16}{4 \times \pi^2 \times .36^2}\ Q^2 = 7.969\ Q^2$$

Substituting these values in equation (*ii*), we get

$$0.3 = 31.87\ Q^2 + 7.969\ Q^2 = 39.839\ Q^2$$

$$\therefore \quad Q = \sqrt{\frac{0.3}{39.839}} = 0.0867\ \text{m}^3/\text{s}$$

$\therefore$ Increase in discharge = $Q - Q^*$ = 0.0867 – 0.0685 = **0.0182 m³/s. Ans.**

Problem 11.34 *A pumping plant forces water through a 600 mm diameter main, the friction head being 27 m. In order to reduce the power consumption, it is proposed to lay another main of appropriate diameter along the side of the existing one, so that two pipes may work in parallel for the entire length and reduce the friction head to 9.6 m only. Find the diameter of the new main if, with the exception of diameter, it is similar to the existing one in every respect.* (A.M.I.E., Winter, 1974)

Solution. Given :

Dia. of single main pipe, d = 600 mm = 0.6 m

Friction head, h_f = 27 m

Friction head for two parallel pipes = 9.6 m

1st Case.

For a single main [Fig. 11.19 (*a*)]

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11.12.1 Power Transmitted Through Nozzle. The kinetic energy of the jet at the outlet of nozzle $= \frac{1}{2} mv^2$

Now mass of liquid at the outlet of nozzle per second $= \rho av$

$\therefore$ Kinetic energy of the jet at the outlet per sec. $= \frac{1}{2} \rho av \times v^2 = \frac{1}{2} \rho av^3$

$\therefore$ Power in kW at the outlet of nozzle $= (\text{K.E./sec}) \times \frac{1}{1000} = \frac{\frac{1}{2}\rho av^3}{1000}$

$\therefore$ Efficiency of power transmission through nozzle,

$$\eta = \frac{\text{Power at outlet of nozzele}}{\text{Power at the inlet of pipe}} = \frac{\frac{\frac{1}{2}\rho av^3}{1000}}{\frac{\rho g . Q . H}{1000}}$$

$$= \frac{\frac{1}{2}\rho av.v^2}{\rho g . Q . H} = \frac{\frac{1}{2}\rho av . v^2}{\rho g . av . H} \qquad \{\because Q = av\}$$

$$= \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}\right] \qquad \ldots(11.26)$$

$$\left(\because \text{ From equation (11.25), } \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D}\frac{a^2}{A^2}}\right]\right)$$

11.12.2 Condition for Maximum Power Transmitted Through Nozzle. We know that, The total head at inlet of pipe = total head at the outlet of the nozzle + losses

i.e.,

$$H = \frac{v^2}{2g} + h_f \qquad \left[\begin{array}{l}\because \text{total head at outlet of nozzle} = \frac{v^2}{2g} \text{ and} \\ h_f = \frac{4 . f . L . V^2}{D \times 2g} = \text{loss of liqui in pipe}\end{array}\right]$$

$$= \frac{v^2}{2g} + \frac{4 . f . L . V^2}{D \times 2g}$$

$$\therefore \qquad \frac{v^2}{2g} = \left(H - \frac{4 . f . L . V^2}{D \times 2g}\right)$$

But power, transmitted through nozzle $= \frac{\frac{1}{2}\rho av^3}{1000} = \frac{\frac{1}{2}\rho av}{1000} \times v^2 = \frac{\frac{1}{2}\rho av}{1000}\left[2g\left(H - \frac{4 . f . L . V^2}{D \times 2g}\right)\right]$

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3. An oil of Kinematic Viscosity 0.5 stoke is flowing through a pipe of diameter 300 mm at the rate of 320 litres per sec. Find the head lost due to friction for a length of 60 m of the pipe. **[Ans.** 5.14 m]
4. Calculate the rate of flow of water through a pipe of diameter 300 mm, when the difference of pressure head between the two ends of a pipe 400 m apart is 5 m of water. Take the value of $f = .009$ in the formula

$$h_f = \frac{4fLV^2}{d \times 2g}$$

[Ans. 0.101 m^3/s]
5. The discharge through a pipe is 200 litres/s. Find the loss of head when the pipe is suddenly enlarged from 150 mm to 300 mm diameter. **[Ans.** 3.672 m]
6. The rate of flow of water through a horizontal pipe is 0.3 m^3/s. The diameter of the pipe is suddenly enlarged from 250 mm to 500 mm. The pressure intensity in the smaller pipe is 13.734 N/cm^2. Determine : (*i*) loss of head due to sudden enlargement, (*ii*) pressure intensity in the large pipe and (*iii*) power lost due to enlargement. **[Ans.** (*i*) 1.07 m, (*ii*) 14.43 N/cm^2, (*iii*) 3.15 kW]
7. A horizontal pipe of diameter 400 mm is suddenly contracted to a diameter of 200 mm. The pressure intensities in the large and smaller pipe is given as 14.715 N/cm^2 and 12.753 N/cm^2 respectively. If $C_c = 0.62$, find the loss of head due to contraction. Also determine the rate of flow of water. **[Ans.** (*i*) 0.571 m, (*ii*) 171. 7 litres/s]
8. Water is flowing through a horizontal pipe of diameter 300 mm at a velocity of 4 m/s. A circular solid plate of diameter 200 mm is placed in the pipe to obstruct the flow. If $C_c = 0.62$, find the loss of head due to obstruction in the pipe. **[Ans.** 2.953 m]
9. Determine the rate of flow of water through a pipe of diameter 10 cm and length 60 cm when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The height of water in the tank from the centre of the pipe is 5 cm. Pipe is given as horizontal and value of $f = .01$. Consider minor losses. **[Ans.** 15.4 litres/s]
10. A horizontal pipe-line 50 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 30 m of its length from the tank, the pipe is 200 mm diameter and its diameter is suddenly enlarged to 400 mm. The height of water level in the tank is 10 m above the centre of the pipe. Considering all minor losses, determine the rate of flow. Take $f = .01$ for both sections of the pipe. **[Ans.** 164.13 litres/s]
11. Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 400 mm and length 500 m. The rate of flow of water through the pipe is 200 litres/s. Consider all losses and take the value of $f = .009$. **[Ans.** 11.79 m]
12. For the problems 9, 10 and 11 draw the hydraulic gradient lines (H.G.L.) and total energy lines (T.E.L.)
13. A syphon of diameter 150 mm connects two reservoirs having a difference in elevation of 15 m. The length of the syphon is 400 m and summit is 4.0 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 80 m. Determine the discharge through the syphon and also pressure at the summit. Neglect minor losses. The co-efficient of friction, $f = .005$. **[Ans.** 41.52 litres/s, – 7.281 m of water]
14. A syphon of diameter 200 mm connects two reservoirs having a difference in elevation as 20 m. The total length of the syphon is 800 m and the summit is 5 m above the water level in the upper reservoir. If the separation takes place at 2.8 m of water absolute find the maximum length of syphon from upper reservoir to the summit. Take $f = .004$ and atmospheric pressure = 10.3 m of water. **[Ans.** 87.52 m]
15. Three pipes of lengths 800 m, 600 m and 300 m and of diameters 400 mm, 300 mm and 200 mm respectively are connected in series. The ends of the compound pipe is connected to two tanks, whose water surface levels are maintained at a difference of 15 m. Determine the rate of flow of water through the pipes if $f = .005$. What will be diameter of a single pipes of length 1700 m and $f = .005$, which replaces the three pipes. **[Ans.** 0.0848 m^3/s, 266.5 mm]

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13

CHAPTER

Boundary Layer Flow

▶ 13.1 INTRODUCTION

When a real fluid flows past a solid body or a solid wall, the fluid particles adhere to the boundary and condition of no slip occurs. This means that the velocity of fluid close to the boundary will be same as that of the boundary. If the boundary is stationary, the velocity of fluid at the boundary will be zero. Farther away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist. The velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary. This variation of velocity from zero to free-stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary. This narrow region of the fluid is called boundary layer. The theory dealing with boundary layer flows is called boundary layer theory.

According to boundary layer theory, the flow of fluid in the neighbourhood of the solid boundary may be divided into two regions as shown in Fig. 13.1.

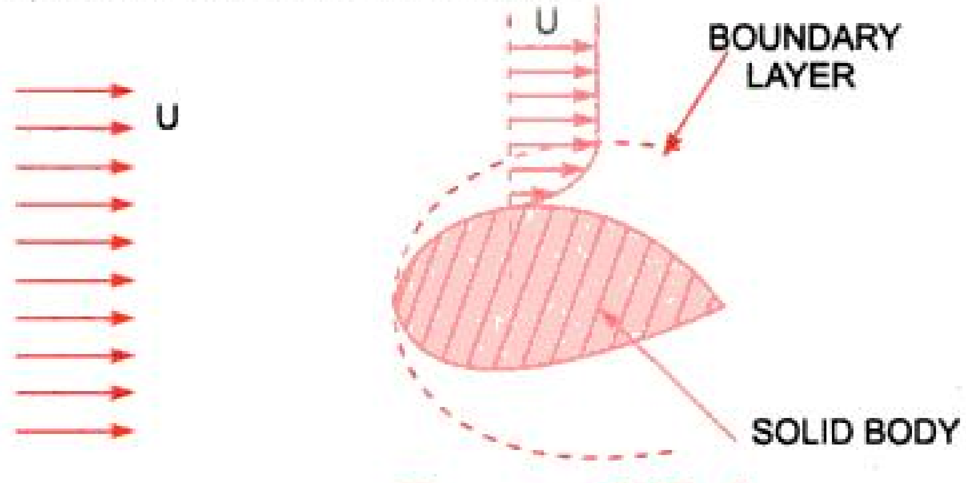

Fig. 13.1 *Flow over solid body.*

1. A very thin layer of the fluid, called the boundary layer, in the immediate neighbourhood of the solid boundary, where the variation of velocity from zero at the solid boundary to free stream velocity in the direction normal to the boundary takes place. In this region, the velocity gradient $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall in the direction of motion. The value of shear stress is given by

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14.2.1 Drag. The component of the total force (F_R) in the direction of motion is called 'drag'. This component is denoted by F_D. Thus drag is the force exerted by the fluid in the direction of motion.

14.2.2 Lift. The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as 'lift'. This is denoted by F_L. Thus lift is the force exerted by the fluid in the direction perpendicular to the direction of motion. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

If the fluid is assumed ideal and the body is symmetrical such as a sphere or cylinder, both the drag and lift will be zero.

▶ 14.3 EXPRESSION FOR DRAG AND LIFT

Consider an arbitrary shaped solid body placed in a real fluid, which is flowing with a uniform velocity U in a horizontal direction as shown in Fig. 14.2. Consider a small elemental area dA on the surface of the body. The forces acting on the surface area dA are :

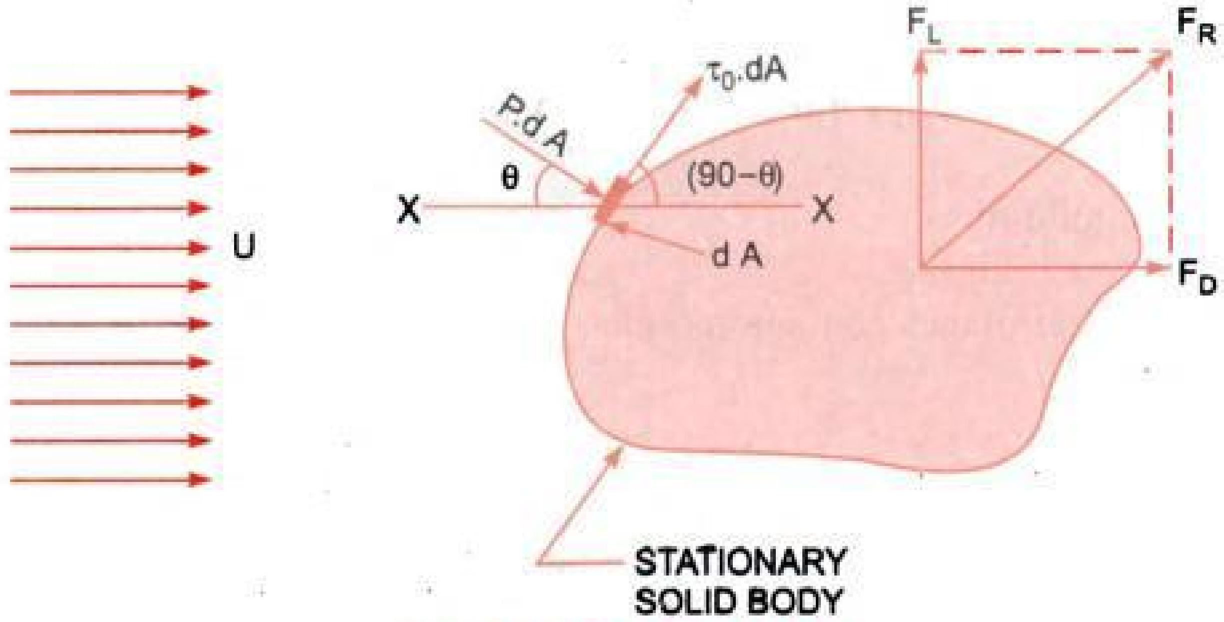

Fig. 14.2 *Drag and lift.*

1. Pressure force equal to $p \times dA$, acting perpendicular to the surface and
2. Shear force equal to $\tau_0 \times dA$, acting along the tangential direction to the surface.

Let θ = Angle made by pressure force with horizontal direction.

(*a*) **Drag Force (F_D).** The drag force on elemental area

= Force due to pressure in the direction of fluid motion

+ force due to shear stress in the direction of fluid motion.

$= pdA \cos\theta + \tau_0 dA \cos(90° - \theta) = pdA \cos\theta + \tau_0 dA \sin\theta$

$\therefore$ Total drag, $\quad F_D$ = Summation of $pdA \cos\theta$ + Summation of $\tau_0 dA \sin\theta$

$$= \int p \cos\theta dA + \int \tau_0 \sin\theta dA. \qquad \text{...(14.1)}$$

The term $\int p \cos\theta dA$ is called the pressure drag or form drag while the term $\int \tau_0 \sin\theta dA$ is called the friction drag or skin drag or shear drag.

(*b*) **Lift Force (F_L).** The lift force an elemental area

= Force due to pressure in the direction perpendicular to the direction of motion

+ Force due to shear stress in the direction perpendicular to the direction of motion.

Problem 14.3 *Find the difference in drag force exerted on a flat plate of size 2 m × 2 m when the plate is moving at a speed of 4 m/s normal to its plane in : (i) water, (ii) air of density 1.24 kg/m³. Co-efficient of drag is given as 1.15.*

Solution. Given :

Area of plate, $A = 2 \times 2 = 4 \text{ m}^2$

Velocity of plate, $U = 4$ m/s

Co-efficient of drag, $C_D = 1.15$

(*i*) Drag force when the plate is moving in water.

Using equation (14.3), $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$, where ρ for water = 1000

$$= 1.15 \times 4 \times 1000 \times \frac{4^2}{2} \text{ N} = 36800 \text{ N}. \quad ...(i)$$

(*ii*) Drag force when the plate is moving in air,

$F_D = C_D \times A \times \dfrac{\rho U^2}{2}$, where ρ for air = 1.24

$$\therefore \quad F_D = 1.15 \times 4.0 \times 1.24 \times \frac{4.0^2}{2.0} \text{ N} = 45.6 \text{ N} \quad ...(ii)$$

∴ Difference in drag force $= (i) - (ii)$

$= 36800 - 45.6 =$ **36754. 4 N. Ans.**

Problem 14.4 *A truck having a projected area of 6.5 square metres travelling at 70 km/hour has a total resistance of 2000 N. Of this 20 per cent is due to rolling friction and 10 per cent is due to surface friction. The rest is due to form drag. Calculate the co-efficient of form drag. Take density of air = 1.25 kg/m³.* (A.M.I.E., Winter, 1976)

Solution. Given :

Area of truck, $A = 6.5 \text{ m}^2$

Speed of truck, $U = 70 \text{ km/hr} = \dfrac{70 \times 100}{60 \times 60} = 19.44$ m/s

Total resistance, $F_T = 2000$ N

Rolling friction resistance, $F_C = 20\%$ of total resistance $= \dfrac{20}{100} \times 2000 = 400$ N

Surface friction resistance, $F_S = 10\%$ of total resistance $= \dfrac{10}{100} \times 2000 = 200$ N

∴ Form drag, $F_D = 2000 - F_C - F_S = 2000 - 400 - 200 = 1400$ N

Using equation (14.3), $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$

where if F_D = Form drag then C_D = Co-efficient of form drag

$$\therefore \quad 1400 = C_D \times 6.5 \times 1.25 \times \frac{19.44^2}{2} \quad (\rho = \text{Density of air} = 1.25 \text{ kg/m}^3)$$

$$\therefore \quad C_D = \frac{1400 \times 2}{6.5 \times 1.25 \times 19.44 \times 19.44} = \textbf{0.912. Ans.}$$

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Using equation (14.11), we get $W = F_D + F_B$

or $$2.794 = 0.2825\ U_1^2 + 0.3286$$

or $$U_1^2 = \frac{2.794 - 0.3286}{0.2825} = 8.725$$

$\therefore$ $$U_1 = \sqrt{8.725} = \mathbf{2.953\ m/s.\ Ans.}$$

When ball is dropped in air. Let the terminal velocity = U_2

Weight, $$W = 2.794$$

Buoyant force, $$F_B = \text{Density of air} \times g \times \text{Volume of ball}$$

$$= 1.25 \times 9.81 \times \frac{\pi}{6}(.04)^3 = 0.000411 \text{ N}$$

Drag force, $$F_D = C_D \times A \times \frac{\rho U^2}{2}, \quad \text{where } \rho \text{ for air} = 1.25$$

$$F_D = 0.1 \times \frac{\pi}{4}(.04)^2 \times 1.25\ \frac{U_2^2}{2} = 0.0000785\ U_2^2.$$

The buoyant force in air is 0.000411, while weight of the ball is 2.794 N. Hence buoyant force is negligible.

$\therefore$ For equilibrium of the ball in air, F_D = Weight of ball

or $$0.0000785\ U_2^2 = 2.794 \text{ or } U_2 = \sqrt{\frac{2.794}{0.0000785}} = 188.67 \text{ m/s}$$

$\therefore$ Increase in terminal velocity in air = $U_2 - U_1$ = 188.67 – 2.9533 = **185.717 m/s. Ans.**

Problem 14.19 *A metallic ball of diameter 2×10^{-3} m drops in a fluid of sp. gr. 0.95 and viscosity 15 poise. The density of the metallic ball is 12000 kg/m^3. Find :*

(i) The drag force exerted by fluid on metallic ball,

(ii) The pressure drag and skin friction drag,

(iii) The terminal velocity of ball in fluid.

Solution. Given :

Diameter of metallic ball, $D = 2 \times 10^{-3}$ m

Sp. gr. of fluid, $S_0 = 0.95$

$\therefore$ Density of fluid, $\rho_0 = 0.95 \times 1000 = 950$ kg/m^3

Viscosity of fluid, $\mu = 15 \text{ poise} = \frac{15}{10} = 1.5 \frac{\text{Ns}}{\text{m}^2}$

Density of ball, $\rho_s = 12000$ kg/m^3

The forces acting on the ball are :

Weight of ball, $$W = \text{Density of ball} \times g \times \text{Volume of ball}$$

$$= 12000 \times 9.81 \times \frac{\pi}{6} D^3$$

$$= 12000 \times 9.81 \times \frac{\pi}{6} \times (2 \times 10^{-3})^3 \text{ N} = 0.000493 \text{ N}$$

Buoyant force, $$F_B = \text{Density of fluid} \times g \times \text{Volume of ball}$$

$$= 950 \times 9.81 \times \frac{\pi}{6}(2 \times 10^{-3})^3 \text{ N} = 0.000039 \text{ N}$$

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Flow over Cylinder due to Constant Circulation. The flow pattern over a cylinder to which a constant circulation (Γ) is imparted is obtained by combining the flow patterns shown in Fig. 14.9 and Fig. 14.11 (*a*). The resultant flow pattern is shown in Fig. 14.12. The velocity at any point on the surface of the cylinder is obtained by adding equations (14.12) and (14.14) as

$$u = u_\theta + u_{\theta_1} = 2u \sin\theta + \frac{\Gamma}{2\pi R}. \qquad \text{...(14.15)}$$

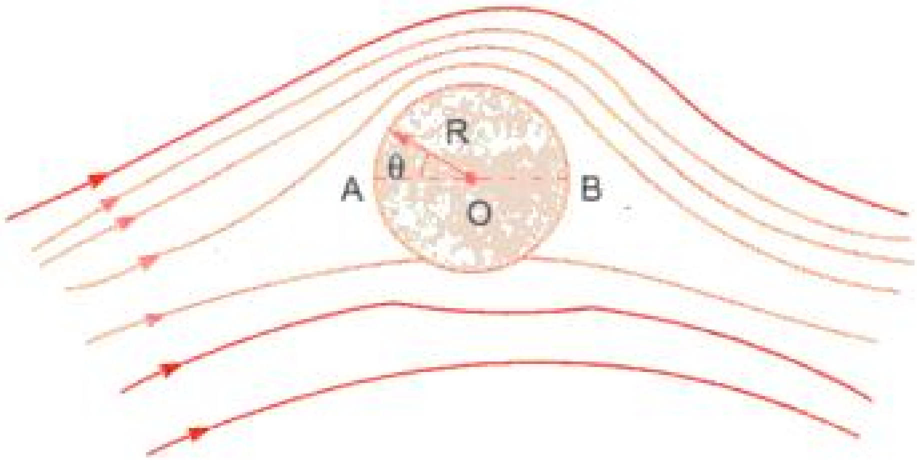

Fig. 14.12 *Flow pattern over a rotating cylinder.*

For the upper half portion of the cylinder, θ varies from 0° to 180° and hence component of velocity, $2U \sin\theta$ is positive. But for the lower half portion of the cylinder, θ varies from 180° to 360°. As $\sin\theta$ for the values of θ more than 180° and less than 360° is negative and hence component of velocity $2U \sin\theta$ will be negative. This means, the velocity on the upper half portion of the cylinder will be more than the velocity on the lower half portion of the cylinder. But from Bernoulli's theorem we know that at a surface where velocity is less pressure will be more there and *vice-versa*. Hence on the lower half portion of cylinder, where velocity is less pressure will be more than the pressure on the upper half portion of the cylinder. Due to this difference of pressure on the two portions of the cylinder, a force will be acting of the cylinder in a direction perpendicular to the direction of flow. This force is nothing but a lift force. Thus by rotating a cylinder at constant velocity in a uniform flow field, a lift force can be developed.

14.7.3 Expression for Lift Force Acting on Rotating Cylinder.

Let a cylinder is rotating is a uniform flow field. The resultant flow pattern will be as shown in Fig. 14.12. Consider a small length of the element of the surface of the cylinder.

Let p_s = Pressure on the surface of the element on cylinder

d_s = Length of element

R = Radius of cylinder

$d\theta$ = Angle made by the length ds at the centre of the cylinder as shown in Fig. 14.13.

p = Pressure of the fluid far away from the cylinder

U = Velocity of fluid far away from the cylinder

u_s = Velocity of fluid on the surface of the cylinder.

Applying Bernoulli's equation to a point far away from cylinder and to a point lying on the surface of cylinder such that both the points are on the same horizontal line, we have

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where V = Mean velocity of flow

D = Hydraulic depth of channel and is equal to the ratio of wetted area to the top width of channel

$= \frac{A}{T}$, where T = Top width of channel.

Sub-critical flow is also called tranquil or streaming flow. For sub-critical flow, $F_e < 1.0$.

The flow is called critical if $F_e = 1.0$. And if $F_e > 1.0$ the flow is called super critical or shooting or rapid or torrential.

▶ 16.3 DISCHARGE THROUGH OPEN CHANNEL BY CHEZY'S FORMULA

Consider uniform flow of water in a channel as shown in Fig. 16.2. As the flow is uniform, it means the velocity, depth of flow and area of flow will be constant for a given length of the channel. Consider sections 1-1 and 2-2.

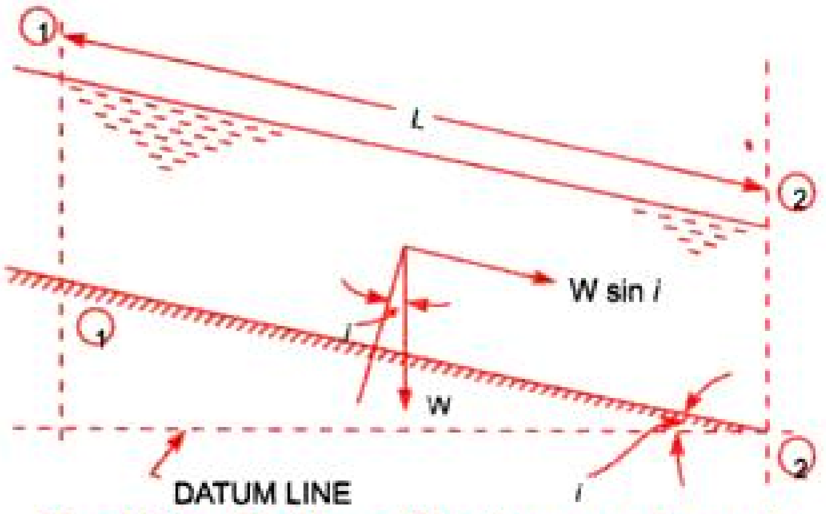

Fig. 16.2 *Uniform flow in open channel.*

Let

L = Length of channel,
A = Area of flow of water,
i = Slope of the bed,
V = Mean velocity of flow of water,
P = Wetted perimeter of the cross-section,
f = Frictional resistance per unit velocity per unit area.

The weight of water between sections 1-1 and 2-2.

W = Specific weight of water × volume of water

$= w \times A \times L$

Component of W along direction of flow $= W \times \sin i = wAL \sin i$ ...(*i*)

Frictional resistance against motion of water $= f \times$ surface area $\times$ (velocity $)^n$

The value of n is found experimentally equal to 2 and surface area $= P \times L$

$\therefore$ Frictional resistance against motion $= f \times P \times L \times V^2$ ...(*ii*)

The forces acting on the water between sections 1-1 and 2-2 are:

1. Component of weight of water along the direction of flow,
2. Friction resistance against flow of water,
3. Pressure force at section 1-1,
4. Pressure force at section 2-2.

As the depths of water at the sections 1-1 and 2-2 are the same the pressure forces on these two sections are same and acting in the opposite direction. Hence they channel each other. In case of uniform flow, the velocity of flow is constant for the given length of the channel. Hence there is no acceleration acting on the water. Hence the resultant force acting in the direction of flow must be zero.

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where $\rho = 1000$ ($\because$ g is given as 10 m/s^2)

$$a = \frac{\pi}{4}(0.05)^2;\ V = 25 \text{ m/s};$$
$$V_{1x} = V\cos 30° = 25\cos 30°,$$
$$V_{2x} = V\cos 80° = 25\cos 80°.$$

Substituting these values in equation (*i*), we get

$$F_x = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25\,[25\cos 30° - 25\cos 80°] = 849.7 \text{ N}$$

The force in the direction of y is given by,

$$F_y = \rho a V[V_{1y} - V_{2y}]$$
$$= 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\sin 30° - 25\sin 80°] = -594.9 \text{ N}$$

The – ve sign shows that force F_y is acting in the downward direction.

The resultant force is given by,

$$F_R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{849.7^2 + 594.9^2} = \mathbf{1037\ N.\ \ Ans.}$$

And the angle made by the resultant with the horizontal is given by,

$$\tan\alpha = \frac{F_y}{F_x} = \frac{594.9}{849.7} = 0.7$$

$\therefore$ $\alpha = \tan^{-1} 0.7 = \mathbf{35°}$. **Ans.**

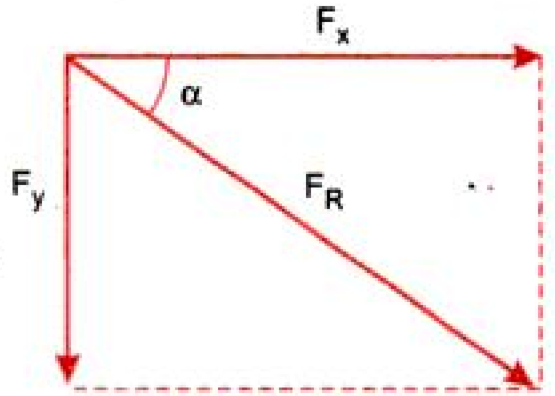

Fig. 17.14 (*b*)

17.4.4 Force Exerted by a Jet of Water on an Un-Symmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the Tips. Fig. 17.15 shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

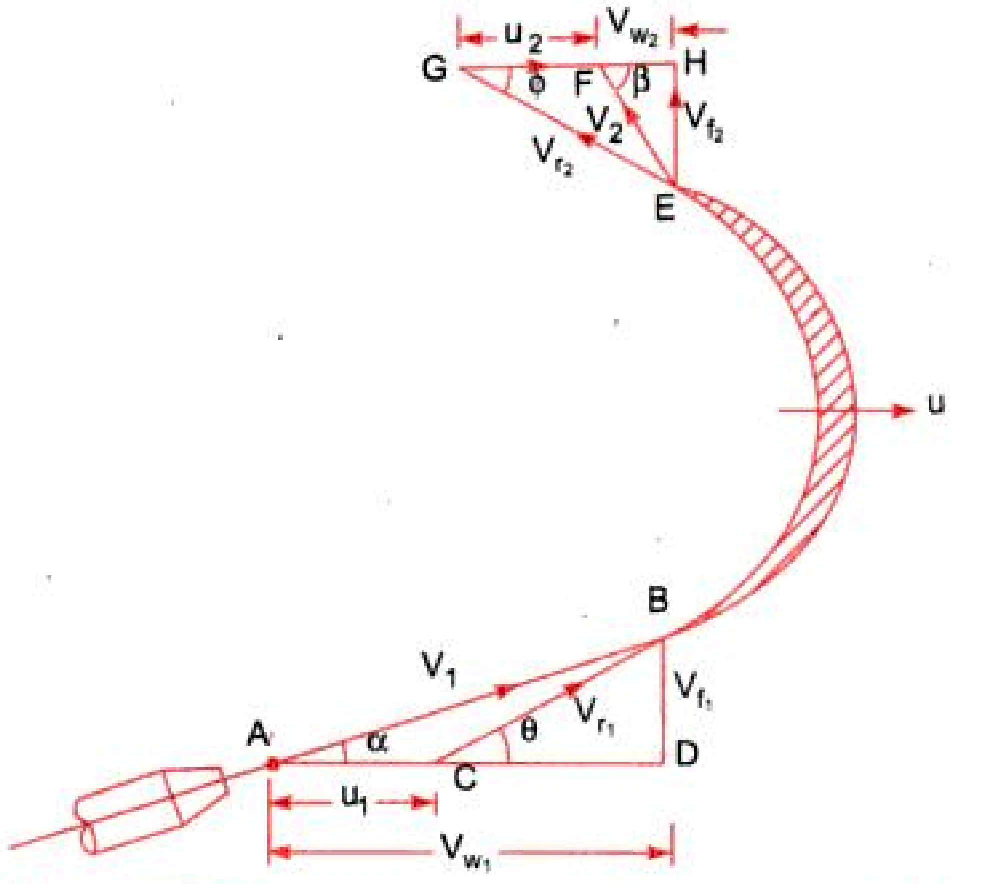

Fig. 17.15 *Jet striking a moving curved vane at one of the tips.*

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where R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g} \frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...for Pelton Turbine}$$

$$= \frac{W}{g} \frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} \text{ kW} \qquad \text{...for a radial Flow Turbine}$$

W.P. = Power supplied at inlet of turbine and also called water power

$$= \frac{W \times H}{1000} \text{ kW} \qquad \text{...(18.3)}$$

where W = Weight of water striking the vannes of the turbine per second

$= \rho g \times Q$ in which Q = Volume of water/s

V_{w_1} = Velocity of whirl at inlet

V_{w_2} = Velocity of whirl at outlet

u = Tangential velocity of vane

u_1 = Tangential velocity of vane at inlet for radial vane

u_2 = Tangential velocity of vane at outlet for radial vane

H = Net head on the turbine.

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW} \qquad \text{...(18.3}A\text{)}$$

For water $\rho = 1000 \text{ kg/m}^3$

$$\therefore \qquad \text{W.P.} = \frac{1000 \times g \times Q \times H}{1000} = g \times Q \times H \text{ kW} \qquad \text{...(18.3}B\text{)}$$

The relation (18.3*B*) is only used when the flowing fluid is water. If the flowing fluid is other than the water, then relation (18.3*A*) is used.

(*b*) **Mechanical Efficiency (η_m).** The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine. The ratio of the power available at the shaft of the turbine (known as S.P. or B.P.) to the power delivered to the runner is defined as mechanical efficiency. Hence, mathematically, it is written as

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}} \qquad \text{...(18.4)}$$

(*c*) **Volumetric Efficiency (η_v).** The volume of the water striking the runner of a turbine is slightly less than the vloume of the water supplied to the turbine. Some of the volume of the water is discharged to the tail race without striking the runner of the turbine. Thus the ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency. It is written as

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}} \qquad \text{...(18.5)}$$

4. Breaking Jet. When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

18.6.1 Velocity Triangles and Work done for Pelton Wheel.

Fig. 18.5 shows the shape of the vanes or buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. Fig. 18.5 (*b*) shows the section of the bucket at *z-z*. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at the outer edge of the bucket, by the same method as explained in Chapter 17.

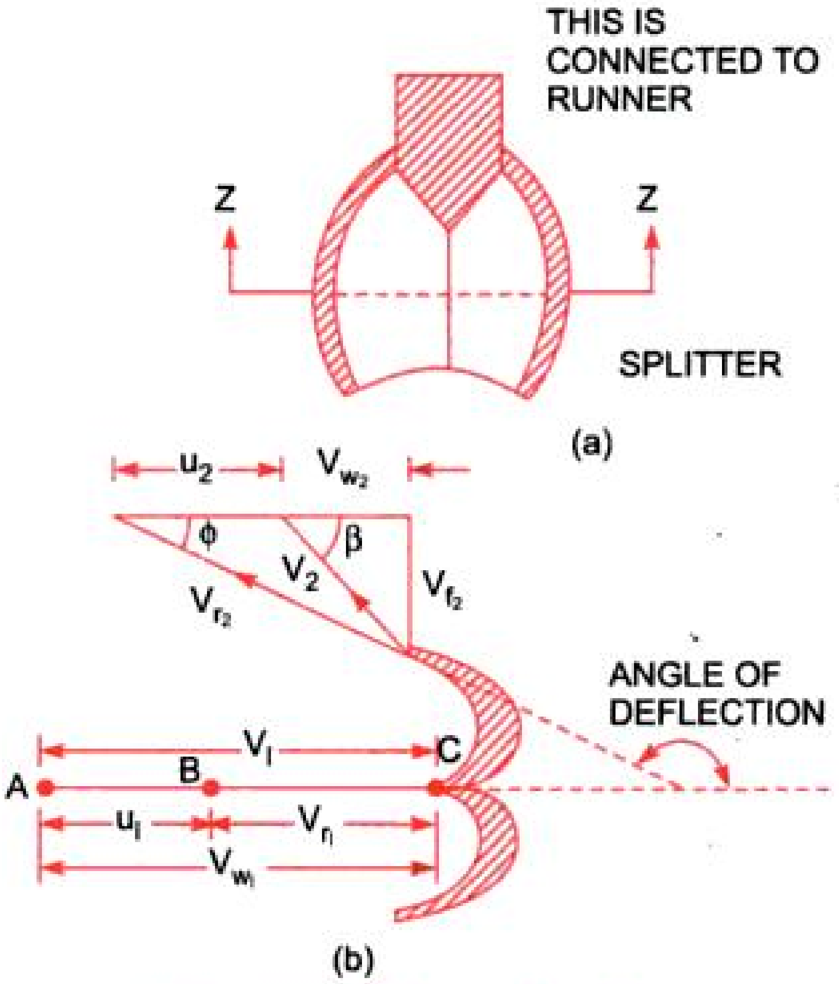

Fig. 18.5 *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet $= \sqrt{2gH}$ ...(18.7)

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0 \text{ and } \theta = 0$$

$\therefore$ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0\ \text{m}^3/\text{s}$

Angle of deflection $= 165°$

$\therefore$ Angle, $\phi = 180 - 165 = 15°$

Speed ratio $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

$\therefore$ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $V_{w_1} = V_1 = 80.86$ m/s

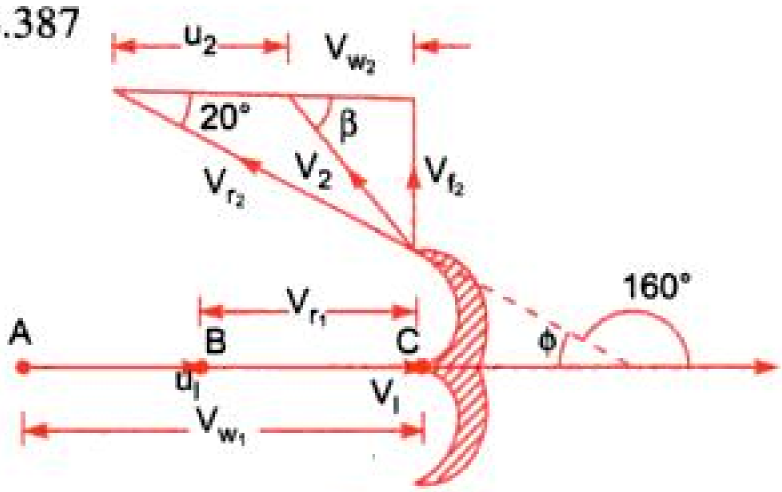

Fig. 18.7

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2}$$

or $44.473 \cos 15 = 36.387 + V_{w_2}$

or $V_{w_2} = 44.473 \cos 15 - 36.387 = 6.57$ m/s.

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u \qquad (\because\ aV_1 = Q)$$

$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$

$\therefore$ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \mathbf{6362.63\ kW.\ Ans.}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$= 0.9731$ or **97.31%. Ans.**

Problem 18.4 *A Pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 r.m.p. The net head on the Pelton wheel is 700 m. If the side clearance angle is 15° and discharge through nozzle is 0.1 m³/s, find :*

(i) Power available at the nozzle, and (ii) Hydraulic efficiency of the turbine.

Solution. Given :

Diameter of wheel, $D = 1.0$ m

Speed of wheel, $N = 1000$ r.p.m.

$\therefore$ Tangential velocity of the wheel, $u = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1.0 \times 1000}{60} = 52.36$ m/s

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Overall efficiency, $\eta_o = 0.80$

Value of $C_v = 0.98$

Speed ratio $= 0.46$

Frequency, $f = 50$ hertz/sec

Now using equation (18.6 A), $\eta_o = \dfrac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)}$

where Q = Total discharge through three nozzles and $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad 0.80 = \frac{10000}{\left(\dfrac{1000 \times 9.81 \times Q \times 400}{1000}\right)}$$

$$\therefore \quad Q = \frac{10000}{0.8 \times 9.81 \times 400} = \mathbf{3.18\ m^3/s.\ Ans.}$$

Discharge through one nozzle $= \dfrac{3.18}{3} = 1.06 \text{ m}^3\text{/s.}$

(*i*) **Diameter of the jet (d).**

Discharged through one nozzle = Area of one jet × Velocity

But velocity of jet, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 400} = 87 \text{ m/s}$

$$\therefore \quad 1.06 = \frac{\pi}{4} d^2 \times 87$$

$$\therefore \quad d = \sqrt{\frac{4 \times 1.06}{\pi \times 87}} = 0.125 \text{ m} = \mathbf{125\ mm.\ Ans.}$$

(*ii*) **Total flow in $\mathbf{m^3/s}$** $= 3.18 \text{ m}^3\text{/s.}$

(*iii*) **Force exerted by a jet on the wheel.**

$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}}$$

$$\therefore \quad u_1 = \text{Speed ratio} \times \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75 \text{ m/s}$$

Now $V_{r_1} = V_1 - u_1 = 87 - 40.75 = 46.25 \text{ m/s}$

and $V_{r_2} = 0.95\ V_{r_1} = 0.95 \times 46.25 = 44.0 \text{ m/s}$

$V_{w_1} = V_1 = 87 \text{ m/s}$

$V_{w_2} = V_{r_2} \cos \phi - u_2 = 44 \times \cos 15° - 40.75 \quad (\because \ u_1 = u_2 = 40.75 \text{ m/s})$

$= 1.75 \text{ m/s}$

Force exerted by a single jet on the buckets

$= \rho \times \text{discharge through one jet} \times (V_{w_1} + V_{w_2})$

$= 1000 \times 1.06\ (87 + 1.75) = 94075 \text{ N} =$ **94.075 kN. Ans.**

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From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9$$

$$\therefore \quad \phi = 20.9 \quad \text{or} \quad \mathbf{20° \; 54.4'. \; Ans.}$$

(*v*) Width of runner at outlet, *i.e.*, B_2

From equation (18.21), we have

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad (\because \; \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2})$$

$$\therefore \quad B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \mathbf{400 \text{ mm. Ans.}}$$

(*vi*) Mass of water flowing through the runner per second.

The discharge, $Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3/\text{s}.$

$$\therefore \quad \text{Mass} = \rho \times Q = 1000 \times 1.0178 \text{ kg/s} = \mathbf{1017.8 \text{ kg/s. Ans.}}$$

(*vii*) Head at the inlet of turbine, *i.e.*, *H*.

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1 \pm V_{w_2} u_2) = \frac{1}{g}(V_{w_1} u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad (\because V_2 = V_{f_2})$$

$$= 9.805 + 0.165 = \mathbf{9.97 \text{ m. Ans.}}$$

(*viii*) Power developed, *i.e.*, $P = \frac{\text{Work done per second on runner}}{1000}$

$$= \frac{\rho Q [V_{w_1} u_1]}{1000} \qquad \text{[Using equation (18.18)]}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \mathbf{97.9 \text{ kW. Ans.}}$$

Hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \mathbf{98.34\%. \; Ans.}$$

Problem 18.16 *A rection turbine works at 450 r.p.m. under a head of 120 metres. Its diameter at inlet is 120 cm and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine :*

(a) The volume flow rate, (b) The power developed, and

(c) Hydraulic efficiency.

Assume whirl at outlet to be zero. (A.M.I.E, Fluid Power—Summer, 1979)

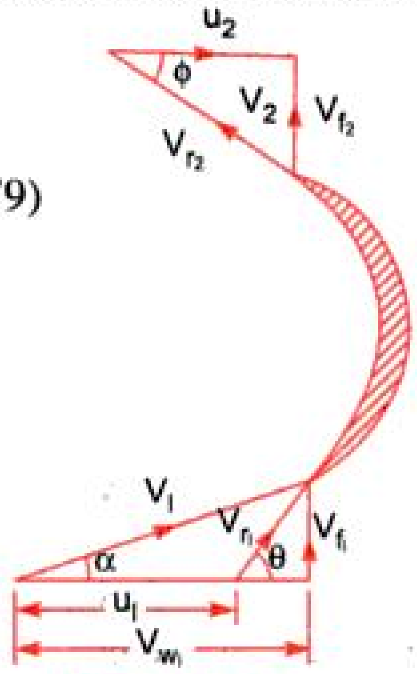

Fig. 18.13

Solution. Given :

Speed of turbine, $N = 450$ r.p.m.

Head, $H = 120$ m

Diameter at inlet, $D_1 = 120 \text{ cm} = 1.2 \text{ m}$

Flow area, $\pi D_1 \times B_1 = 0.4 \text{ m}^2$

Angle made by absolute velocity at inlet, $\alpha = 20°$

Angle made by the relative velocity at inlet, $\theta = 60°$

Whirl at outlet, $V_{w_2} = 0$

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the area of the guide vanes increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water. The water from the guide vanes then passes through the surrounding casing which is in most of the cases concentric with the impeller as shown in Fig. 19.2 (*b*).

3. Suction Pipe with a Foot - valve and a Strainer. A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.

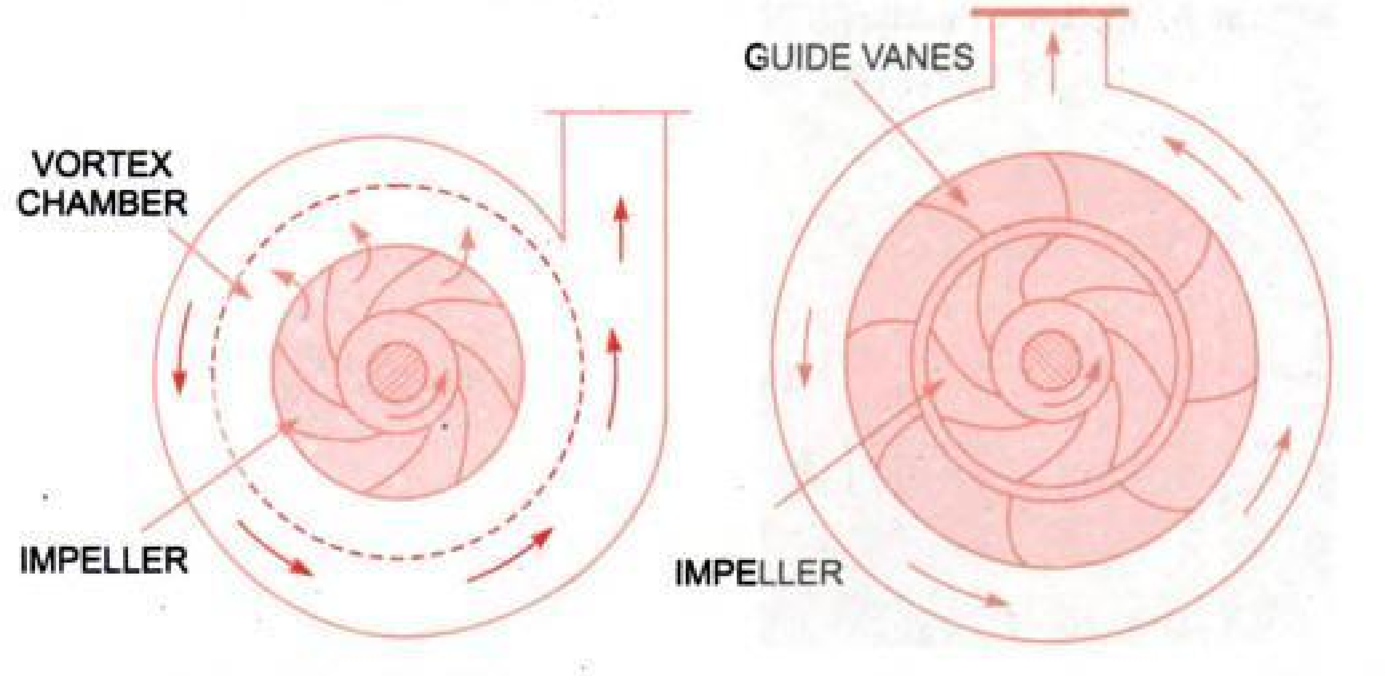

Fig. 19.2 *Different types of casing.*

4. Delivery pipe. A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height is known as delivery pipe.

▶ 19.3 WORK DONE BY THE CENTRIFUGAL PUMP (OR BY IMPFLLER) ON WATER

In case of the centrifugal pump, work is done by the impeller on the water. The expression for the work done by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as for a turbine. The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet. Hence angle $\alpha = 90°$ and $V_{w_1} = 0$. For drawing the velocity triangles, the same notations are used as that for turbines. Fig. 19.3 shows the velocity triangles at the inlet and outlet tips of the vanes fixed to an impeller.

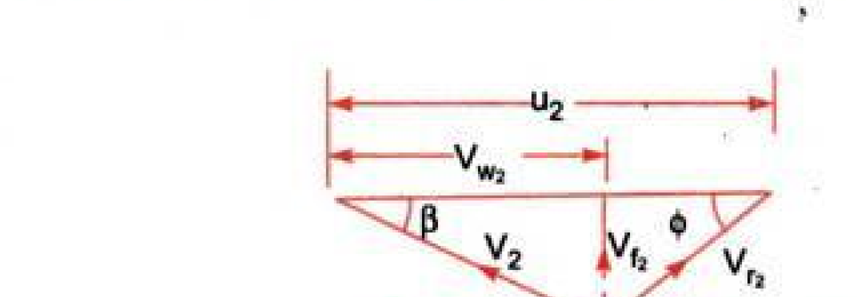

Fig. 19.3 *Velocity triangles at inlet and outlet.*

Let N = Speed of the impeller in r.p.m.,

D_1 = Diameter of impeller at inlet,

u_1 = Tangential velocity of impeller at inlet,

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%. \ Ans.}$$

Problem 19.5 *A centrifugal pump discharges 0.15 m³/s of water against a head of 12.5 m, the speed of the impeller being 600 r.p.m. The outer and inner diameters of impeller are 500 mm and 250 mm respectively and the vanes are bent back at 35° to the tangent at exit. If the area of flow remains 0.07 m² from inlet to outlet, calculate :*

(i) Manometric efficiency of pump, (ii) Vane angle at inlet, and

(iii) Loss of head at inlet to impeller when the discharge is reduced by 40% without changing the speed. (A.M.I.E., Summer, 1988)

Solution. Given :

Discharge, $Q = 0.15$ m³/s

Head, $H_m = 12.5$ m

Speed, $N = 600$ r.p.m.

Outer dia., $D_2 = 500$ mm $= 0.50$ m

Inner dia., $D_1 = 250$ mm $= 0.25$ m

Vane angle at outlet, $\phi = 35°$

Area of flow, $= 0.07$ m²

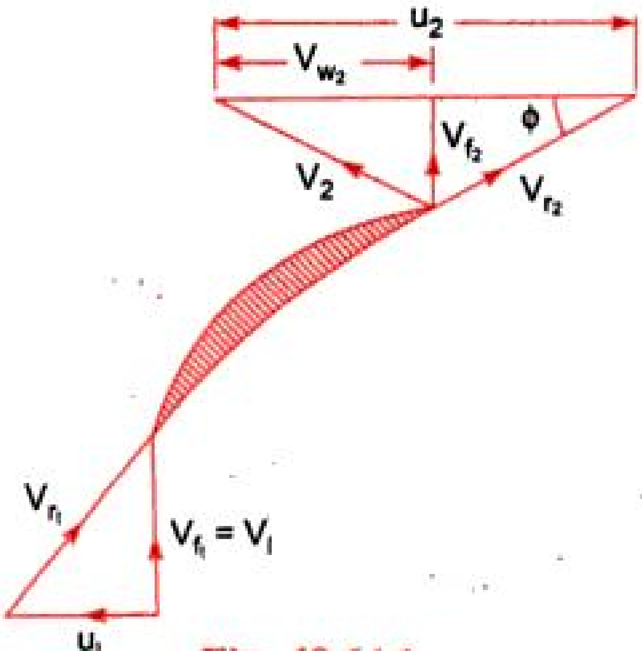

Fig. 19.6(*a*)

As area of flow is constant from inlet to outlet, then velocity of flow will be constant from inlet to outlet.

Dischage = Area of flow × Velocity of flow

or 0.15 = 0.07 × Velocity of flow

$\therefore$ Velocity of flow $= \dfrac{0.15}{0.07} = 2.14$ m/s.

$$\therefore \quad V_{f_1} = V_{f_2} = 2.14 \text{ m/s.}$$

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 600}{60} = 7.85 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 600}{60} = 15.70 \text{ m/s}$$

From outlet velocity triangle, $V_{w_2} = u_2 - \dfrac{V_{f_2}}{\tan\phi} = 15.70 - \dfrac{2.14}{\tan 35°} = 12.64$ m/s

(*i*) *Manometric efficiency of the pump*

Using equation (19.8), we have $\eta_{man} = \dfrac{g \times H_m}{V_{w_2} \times u_2} = \dfrac{9.81 \times 12.5}{12.64 \times 15.7} = 0.618$ or **61.8%. Ans.**

(*ii*) *Vane angle at inlet* (θ)

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{2.14}{7.85} = 0.272$

$$\therefore \quad \theta = \tan^{-1} 0.272 = \mathbf{15° \ 12'. \ Ans.}$$

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Now H_m is given by equation (19.6) as

$$H_m = \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad \text{...(ii)}$$

where $\frac{p_o}{\rho g}$ = Pressure head at outlet of pump = h_d =30 m

$\frac{V_o^2}{2g}$ = Velocity head at outlet of pump = $\frac{V_d^2}{2g}$

$\frac{p_i}{\rho g}$ = Pressure head at inlet of pump = h_s = 6 m

$\frac{V_i^2}{2g}$ = Velocity head at inlet of pump = $\frac{V_s^2}{2g}$

Z_o and Z_i = Vertical height of outlet and inlet of the pump from datum line.

If $Z_o = Z_i$ then equation(*ii*) becomes as

$$H_m = \left(30 + \frac{V_d^2}{2g}\right) - \left(6 + \frac{V_s^2}{2g}\right) \quad \text{...(iii)}$$

Now
$$V_d = \frac{\text{Discharg}}{\text{Area of delivery pipe}} = \frac{0.04}{\frac{\pi}{4}(D_d)^2} = \frac{.04}{\frac{\pi}{4} \times .1^2} = 5.9 \text{ m/s}$$

And
$$V_s = \frac{.04}{\text{Area of suction pipe}} = \frac{.04}{\frac{\pi}{4} D_s^2} = \frac{.04}{\frac{\pi}{4} \times .15^2} = 2.26 \text{ m/s.}$$

Substituting these values in equation (*iii*), we get

$$H_m = \left(30 + \frac{5.09^2}{2 \times 9.81}\right) - \left(6 + \frac{2.26^2}{2 \times 9.81}\right)$$
$$= (30 + 1.32) - (6 + .26) = 31.32 - 6.26 = 25.06 \text{ m.}$$

Substituting the value of 'H_m' in equation (*i*), we get

$$\eta_o = .02424 \times 25.06 = 0.6074 = \mathbf{60.74\%.\ Ans.}$$

(*iii*) **Manometric efficiency of the pump (η_{man}).**

Tangential velocity at outlet is given by

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = 20.94 \text{ m/s.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.0}{20.94 - V_{w_2}}$$

$$\therefore \quad 20.94 - V_{w_2} = \frac{2.0}{\tan \phi} = \frac{2.0}{\tan 45} = 2.0$$

$$\therefore \quad V_{w_2} = 20.94 - 2.0 = 18.94.$$

Using equation (19.8), $\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25.06}{18.94 \times 20.94} = 0.6198 = \mathbf{61.98\%.\ Ans.}$

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19.5 MINIMUM SPEED FOR STARTING A CENTRIFUGAL PUMP

If the pressure rise in the impeller is more than or equal to manometric head (H_m), the centrifugal pump will start delivering water. Otherwise, the pump will not discharge any water, though the impeller is rotating. When impeller is rotating, the water in contact with the impeller is also rotating. This is the case of forced vortex. In case of forced vortex, the centrifugal head or head due to pressure rise in the impeller.

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} \quad \text{...(i)}$$

where ωr_2 = Tangential velocity of impeller at outlet = u_2 and
ωr_1 = Tangential velocity of impeller at inlet = u_1

$\therefore$ Head due to pressure rise in impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

The flow of water will commence only if

Head due to pressure rise in impeller $\geq H_m$ or $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$.

For minimum speed, we must have
$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m \quad \text{...(19.13)}$$

But from equation (19.8), we have
$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$\therefore \quad H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}.$$

Substituting this value of H_m in equation (19.13),

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g} \quad \text{...(19.14)}$$

Now
$$u_2 = \frac{\pi D_2 N}{60} \text{ and } u_1 = \frac{\pi D_1 N}{60}.$$

Substituting the values of u_2 and u_1 in equation (19.14),

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \times \pi D_2 N}{g \times 60}$$

Dividing by $\frac{\pi N}{g \times 60}$, we get
$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{man} \times V_{w_2} \times D_2$$

or
$$\frac{\pi N}{120}\,[D_2^2 - D_1^2] = \eta_{man} \times V_{w_2} \times D_2$$

$$\therefore \quad N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]} \quad \text{...(19.15)}$$

Equation (19.15) gives the minimum starting speed of the centrifugal pump.

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$$\therefore \quad \frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}} \quad \text{or} \quad \frac{2000 \times \sqrt{3}}{30^{3/4}} = \frac{1500 \times \sqrt{5}}{H_{m_2}^{3/4}}$$

$$\therefore \quad H_{m_2}^{3/4} = \frac{1500 \times \sqrt{5} \times 30^{3/4}}{2000 \times \sqrt{3}} = \frac{1500}{2000} \times \sqrt{\frac{5}{3}} = 12.818 = 12.411$$

$$\therefore \quad H_{m_2} = (12.411)^{4/3} = 28.71 \text{ m}$$

$$\therefore \quad \text{Number of stages} = \frac{\text{Total head}}{\text{Head per stage}} = \frac{20}{28.71} = 6.96 \simeq \textbf{7. Ans.}$$

Using equation (19.20), we have

$$\frac{\sqrt{H_{m_1}}}{D_1 N_1} = \frac{\sqrt{H_{m_2}}}{D_2 N_2} \quad \text{or} \quad \frac{\sqrt{30}}{0.30 \times 2000} = \frac{\sqrt{28.71}}{D_2 \times 1500}$$

$$\therefore \quad D_2 = \frac{0.30 \times 2000 \times \sqrt{28.71}}{1500 \times \sqrt{30}} = 0.3913 \text{ m} = \textbf{391.3 mm. Ans.}$$

Problem 19.19 *Find the number of pumps required to take water from a deep well under a total head of 89 m. All the pumps are identical and are running at 800 r.p.m. The specific speed of each pump is given as 25 while the rated capacity of each pump is 0.16 m^3/s.*

Solution. Given :

Total head $= 89$ m

Speed, $N = 800$ r.p.m.

Specific speed, $N_s = 25$

Rate capacity, $Q = 0.16 \text{ m}^3/\text{s}$

Let H_m = Head developed by each pump.

Using equation (19.18), $N_s = \dfrac{H\sqrt{Q}}{H_m^{3/4}}$

$$25 = \frac{800 \times \sqrt{0.16}}{H_m^{3/4}}$$

$$\therefore \quad H_m^{3/4} = \frac{800 \times \sqrt{0.16}}{25} = 12.8$$

$$\therefore \quad H_m = (12.8)^{4/3} = 29.94 \text{ m}$$

$$\therefore \quad \text{Number of pumps required} = \frac{\text{Total head}}{\text{Head developed by one pump}} = \frac{89}{29.94} \simeq \textbf{3. Ans.}$$

As the total head is more than the head developed by one pump, the pumps should be connected in series.

Problem 19.20 *Two geometrically similar pumps are running at the same speed of 1000 r.p.m. One pump has an impeller diameter of 0.30 metre and lifts water at the rate of 20 litres per second against a head of 15 metres. Determine the head and impeller diameter of the other pump to deliver half the discharge.* (A.M.I.E., Summer, 1980)

Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

$$\frac{p_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L \quad ...(i)$$

where p_a = Atmospheric pressure on the free surface of liquid
V_a = Velocity of liquid at the free surface of liquid $\simeq 0$
Z_a = Height of free surface from datum line = 0
p_1 = Absolute pressure at the inlet of pump
V_1 = Velocity of liquid through suction pipe = v_s
Z_1 = Height of inlet of pump from datum line = h_s
h_L = Loss of head in the foot valve, strainer and suction pipe = h_{f_s}

Hence the above equation *(i)*, after substituting the above values becomes as

$$\frac{p_a}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or
$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or
$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right) \quad ...(ii)$$

For finding the maximum suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get

$p_1 = p_v$, where p_v = vapour pressure of the liquid in absolute units.

Now the equation *(ii)* becomes as

$$\frac{p_v}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

or
$$\frac{p_a}{\rho g} = \frac{p_v}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s} \quad (\because \; p_1 = p_v) \; ...(iii)$$

Now $\frac{p_a}{\rho g}$ = Atmospheric pressure head $= H_a$ (meter of liquid)

$\frac{p_v}{\rho g}$ = Vapour pressure head = H_v (meter of liquid)

Now the equation *(iii)* becomes as

$$H_a = H_v + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or
$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s} \quad ...(19.31)$$

The above equation (19.31) gives the value of maximum suction lift (or maximum suction height)for a centrifugal pump. Hence the suction height of any pump should not be more than that given by equation (19.31). If the suction height of the pump is more, then vapourization of liquid at inlet of pump will take place and there will be a possibility of cavitation.

$$= \frac{1}{g} V_{w_2} \times u_2$$

where V_{w_2} = Velocity of whirl at outlet, and

u_2 = Tangential velocity of wheel at outlet .

3. The vertical height of the centre-line of the centrifugal pump from the water surface in the pump is called the suction head (h_s).
4. Delivery head (h_d) is the vertical distance between the centre-line of the pump and the water surface in the tank to which water is lifted.
5. Manometric head (H_m) is the head against which a centrifugal pump has to work. It is given as

(*a*)
$$H_m = \frac{V_{w_2} \times u_2}{g} - \text{Loss of head in impeller and casing}$$
$$= \frac{V_{w_2} \times u_2}{g} \quad \text{...if losses in pump is zero}$$

(*b*)
$$H_m = \text{Total head at outlet} - \text{Total head at inlet of pump}$$
$$= \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$$

(*c*)
$$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}.$$

6. The efficiencies of a pump are : *(i)* Manometric efficiency (h_{man}), *(ii)* Mechanical efficiency (η_m), and *(iii)* Overall efficiency (η_o) Mathematically they are given as

$$\eta_{\text{man}} = \frac{gH_m}{V_{w_2} \times u_2}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} \times u}{75}\right)}{\text{S.P.}}, \text{ where } W = w \times Q$$

$$\eta_o = \frac{W \times H_m}{1000 \times \text{S.P.}}$$

7. The minimum speed for starting a centrifugal pump is given by $N = \dfrac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}$.
8. If a centrifugal pump consists of two or more impellers, the pump is called a multistage pump. To produce a high head, the impellers are connected in series while to discharge a large quantity of liquid, the impellers are connected in parallel.
9. The specific speed of a centrifugal pump is defined as the speed at which a pump runs when the head developed is one metre and discharge is one cubic metre. Mathematically, it is given as

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}, \text{ where } H_m = \text{Manometric hand.}$$

10. For complete similarity between the model and actual centrifugal pump (prototype) the following conditions should be satisfied :

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1. A cylinder with a piston, piston rod, connecting rod and a crank,
2. Suction pipe,
3. Delivery pipe,
4. Suction valve, and
5. Delivery valve.

▶ 20.3 WORKING OF A RECIPROCATING PUMP

Fig. 20.1 shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at A., the piston is at the extreme left position in the cylinder. As the crank is rotating from A to C, (*i.e.*, from $\theta = 0$ to $\theta = 180°$), the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder. Thus the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

When crank is rotating from C to A (*i.e.*, from $\theta = 180°$ to $\theta = 360°$), the piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

20.3.1 Discharge Through a Reciprocating Pump.

Consider a single* acting reciprocating pump as shown in Fig. 20.1.

Let D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$= \frac{\pi}{4} D^2$$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke = $2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$$= \text{Area} \times \text{Length of stroke} = A \times L$$

Number of revolution per second, $= \dfrac{N}{60}$

∴ Discharge of the pump per second,

$$Q = \text{Discharge in one revolution} \times \text{No. of revolution per second}$$

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \qquad \text{...(20.1)}$$

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* Single acting means the water is acting on one side of the piston only.

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$$W = 3000 \text{ N}, \frac{l}{L} = \frac{1}{10}$$

Let F' = Force required at the end of the lever.

Using equation (21.4), $$W = F' \times \frac{L}{l} \times \frac{A}{a}$$

$\therefore$ $$F' = W \times \frac{l}{L} \times \frac{a}{A} = 3000 \times \frac{1}{10} \times \frac{7.068 \times 10^{-4}}{0.0314} = \textbf{6.752 N. Ans.}$$

Problem 21.4 *If in the problem 21.1, the stroke of the plunger is 100 mm, find the distance travelled by the weight in 100 strokes. Determine the work done during 100 strokes.*

Solution. The data given in problem 21.1 :

$$D = 0.30 \text{ m}, A = 0.07068 \text{ m}^2, d = 0.045 \text{ m}, a = .00159 \text{ m}^2$$
$$F = 50 \text{ N and } W \text{ (calculated)} = 2222.64 \text{ N}$$

Stroke of plunger = 100 mm = 0.10 m

Number of strokes = 100

Volume of liquid displaced by plunger in one stroke

= Area of plunger × Stroke of plunger

$= a \times 0.10 \text{ m}^3 = .00159 \times 0.10 = .000159 \text{ m}^3.$

The liquid displaced by plunger, will enter the cylinder in which ram is fitted and this liquid will move the ram in the upward direction.

Let the distance moved by the ram or weight in one stroke

$= x$ m

Then volume displaced by ram in one stroke

$= \text{Area of ram} \times x = A \times x = 0.07068 \times x \text{ m}^3$

As volume displaced by plunger and ram is the same,

$\therefore$ $$.000159 = .07068 \times x$$

$\therefore$ $$x = \frac{.000159}{.07068} = .00225 \text{ m}$$

$\therefore$ Distance moved by weight in 100 strokes

$= x \times 100 = .00225 \times 100 =$ **0.225 m. Ans.**

Work done during 100 strokes = Weight lifted × Distance moved

$= W \times 0.225 = 2222.64 \times 0.225$ N-m = **500.094 N-m. Ans.**

Problem 21.5 *A hydraulic press has a ram of 150 mm diameter plunger of 20 mm diameter. The stroke of the plunger is 200 mm and weight lifted is 800 N. If the distance moved by the weight is 1.0 m in 20 minutes determine :*

(*i*) *The force applied on the plunger,* (*ii*) *Power required to drive the plunger, and*

(*iii*) *Number of strokes performed by the plunger.*

Solution. Given :

Diameter of ram, $D = 150 \text{ mm} = 0.15 \text{ m}$

Diameter of plunger, $d = 20 \text{ mm} = 0.02 \text{ m}$

Stroke of plunger $= 200 \text{ mm} = 0.20 \text{ m}$

Weight lifted, $W = 800 \text{ N}$

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21.3.2 Differential Hydraulic Accumulator.

It is a device in which the liquid is stored at a high pressure by a comparatively small load on the ram. It consists of a fixed vertical cylinder of small diameter as shown in Fig. 21.5. The fixed vertical cylinder is surrounded by closely fitting brass bush, which is surrounded by an inverted moving cylinder, having circular projected collar at the base on which weights are placed.

The liquid from the pump is supplied to the fixed vertical cylinder. The liquid moves up through the small diameter of fixed vertical cylinder and then enters the inverted cylinder. The water exerts an upward pressure force on the internal annular area of the inverted moving cylinder, which is loaded at the base. The internal annular area of the inverted moving cylinder is equal to the sectional area of the brass bush. When the inverted moving cylinder moves up, the hydraulic energy is stored in the accumulator.

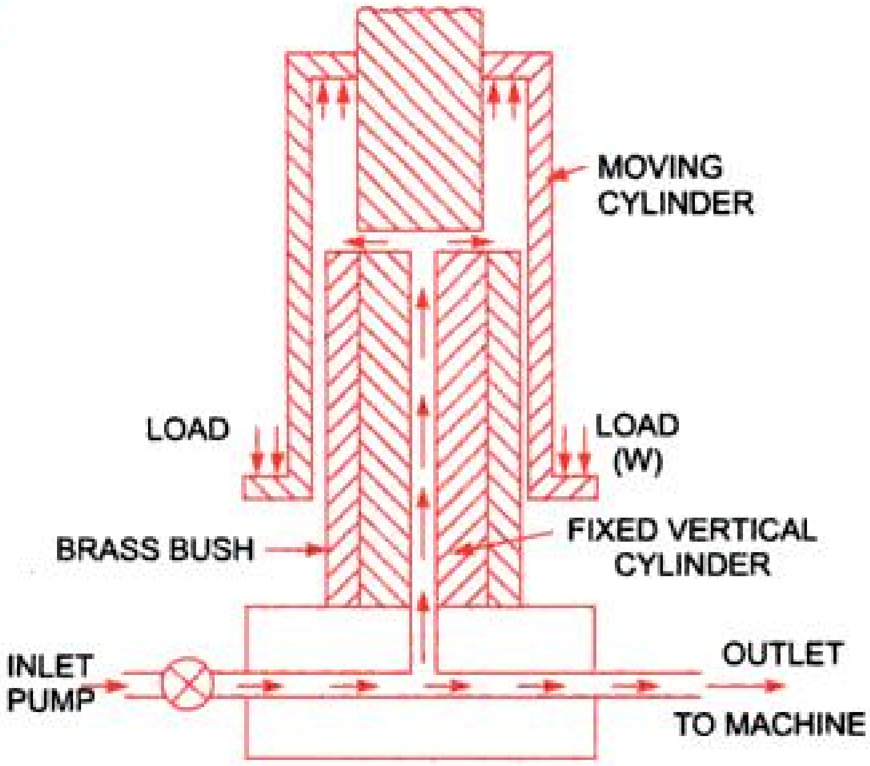

Fig. 21.5 *Differential hydraulic accumulator.*

Let p = Intensity of pressure of liquid supplied by pump,

a = Area of brass-bush,

L = Vertical lift of the moving cylinder,

W = Total weight placed on the moving cylinder including the weight of cylinder.

Then $W = p \times a$

$$\therefore \quad P = \frac{W}{a} \qquad \text{...(21.7)}$$

From equation (21.7), it is clear that pressure intensity can be increased with a small load W, by making area 'a' small.

Now total energy stored in the accumulator = Total weight × Vertical lift

$$= W \times L \text{ Nm.} \qquad \text{...[21.7 (a)]}$$

▶ 21.4 THE HYDRAULIC INTENSIFIER

The device, used to increase the intensity of pressure of water by means of hydraulic energy available from a large amount of water at a low pressure, is called the hydraulic intensifier. Such a device is needed when the hydraulic machines such as hydraulic press requires water at very high pressure which cannot be obtained from the main supply directly.

A hydraulic intensifier consists of fixed ram through which the water, under a high pressure, flows to the machine. A hollow inverted sliding cylinder, containing water under high pressure, is mounted over the fixed ram. The inverted sliding cylinder is surrounded by another fixed inverted cylinder which contains water from the main supply at a low pressure as shown in Fig. 21.6

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(a) more than 10 times the depth of orifice
(b) less than 10 times the depth of orifice
(c) less than 5 times the depth of orifice
(d) none of the above.

46. Which mouthpiece is having maximum co-efficient of discharge
(a) external mouthpiece
(b) convergent- divergent mouthpiece
(c) internal mouthpiece
(d) none of the above.

47. The co-efficient of discharge (C_d)
(a) for an orifice is more than that for a mouthpiece
(b) for internal mouthpiece is more than that for external mouthpiece
(c) for a mouthpiece is more than that for an orifice
(d) none of the above.

48. A flow is said to be laminar when
(a) the fluid particles moves in a zig-zag way
(b) the Reynold number is high
(c) the fluid particles move in layers parallel to the boundary
(d) none of the above.

49. For the laminar flow through a circular pipe
(a) the maximum velocity = 1.5 times the average velocity
(b) the maximum velocity = 2.0 times the average velocity
(c) the maximum velocity = 2.5 times the average velocity
(d) none of the above.

50. The loss of pressure head for the laminar flow through pipes varies
(a) as the square of velocity
(b) directly as the velocity
(c) as the inverse of the velocity
(d) none of the above.

51. For the laminar flow through a pipe, the shear stress over the cross-section
(a) varies inversely as the distance from the centre of the pipe
(b) varies directly as the distance from the surface of the pipe
(c) varies directly as the distance from the centre of the pipe
(d) remains constant over the cross-section.

52. For the laminar flow between two parallel plates
(a) the maximum velocity = 2.0 times the average velocity
(b) the maximum velocity = 2.5 times the average velocity
(c) the maximum velocity = 1.33 times the average velocity
(d) none of the above.

53. The value of the kinetic energy correction factor (α) of the viscous flow through a circular pipe is
(a) 1.33 (b) 1.50
(c) 2.0 (d) 1.25.

54. The value of the momentum correction factor (β) for the viscous flow through a circular pipe is
(a) 1.33 (b) 1.50
(c) 2.0 (d) 1.25.

55. The pressure drop per unit length of a pipe for laminar flow is
(a) equal to $\dfrac{12\mu\bar{U}L}{\rho g D^2}$ (b) equal to $\dfrac{12\mu\bar{U}}{\rho g D^2}$
(c) equal to $\dfrac{32\mu\bar{U}L}{\rho g D^2}$ (d) none of the above.

56. For viscous flow between two parallel plates, the pressure drop per unit length is equal to
(a) $\dfrac{12\mu\bar{U}L}{\rho g D^2}$ (b) $\dfrac{12\mu\bar{U}L}{D^2}$
(c) $\dfrac{32\mu\bar{U}L}{D^2}$ (d) $\dfrac{12\mu\bar{U}}{D^2}$.

57. The velocity distribution in laminar flow through a circular pipe follow the
(a) parabolic law (b) linear law
(c) logarithmic law (d) none of the above.

58. A boundary is known as hydrodynamically smooth if
(a) $\dfrac{k}{\delta'} = 0.3$ (b) $\dfrac{k}{\delta'} > 0.3$
(c) $\dfrac{k}{\delta'} < 0.25$ (d) $\dfrac{k}{\delta'} = 6.0$
where k = Average height of the irregularities from the boundary
and δ' = Thickness of laminar sub-layer.

59. The co-efficient of friction for laminar flow through a circular pipe is given by
(a) $f = \dfrac{0.0791}{(R_e)^{1/4}}$ (b) $f = \dfrac{16}{R_e}$
(c) $f = \dfrac{64}{R_e}$ (d) none of the above.

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